

Heuristic Optimisation of 2000+ dimensional tests

Kalin Penev

School of Media, Art and Technology

Southampton Solent University

Kalin.Penev@solent.ac.uk

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Aims

- **Explore 2000 and 2016 dimensional numerical tests**
- **Investigate**
 - Identification of unknown multidimensional solutions
 - Adaptation to heterogeneous tasks
 - Time for experiments completion
 - Study search process time deviations
- **Identify potential issues & limitations**
 - hardware
 - software

Numerical tests

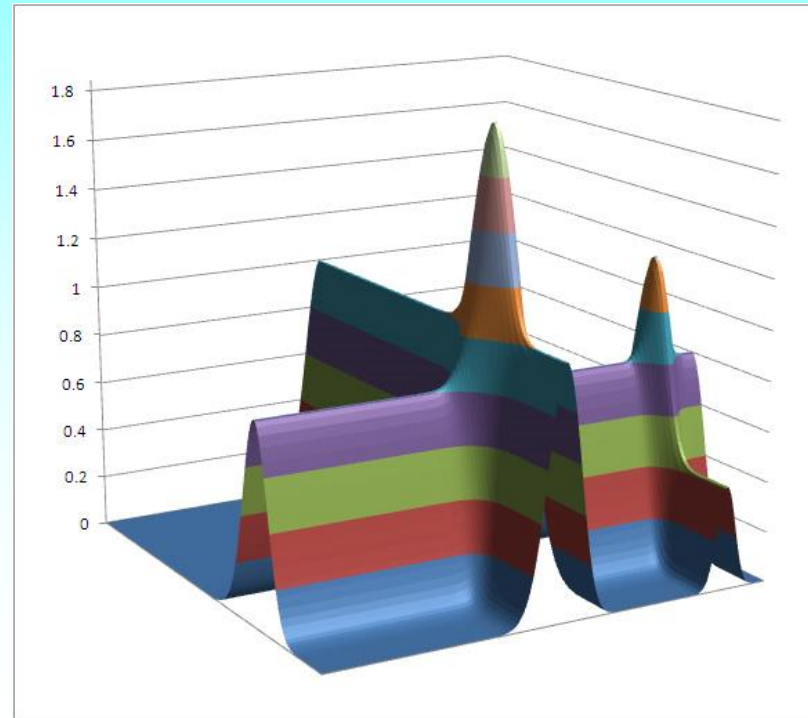
Michalewicz test

$$f(x_1, x_2) = \sum_{i=1}^2 \sin(x_i) (\sin(ix_i^2 / \pi))^{2m}$$

where $x_i \in [0.0, 3.0]$ for $i=1, \dots, n$

Maximum is dependant on dimensions
number.

Maximum is unknown



Numerical tests

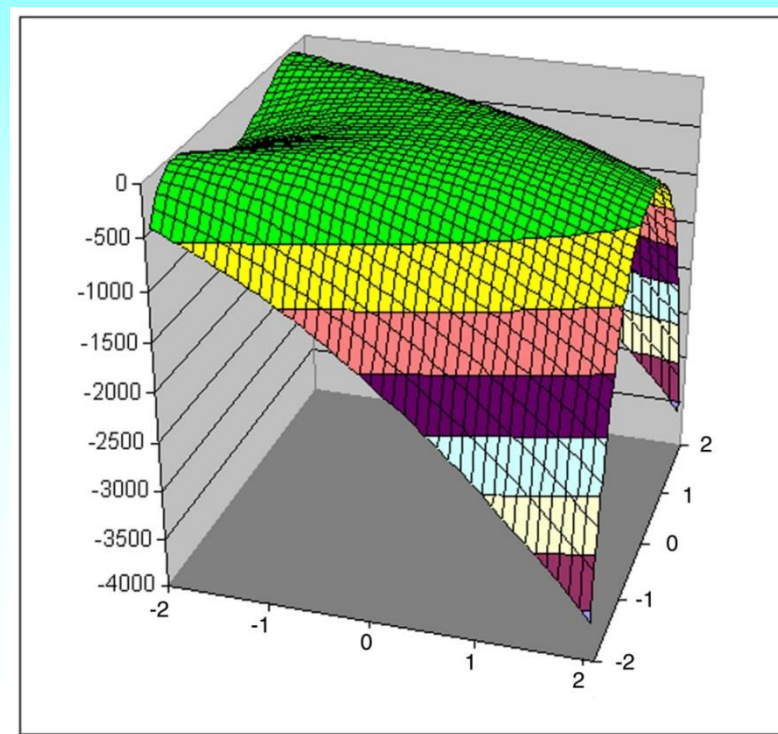
Rosenbrock test

$$f(x_i) = \sum_{i=1}^{n-1} [100 * (x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$$

where $x_i \in [-2.0, 2.0]$

for $i=1, \dots, n-1$,

Maximum is $f(x_i=1) = 0$



Numerical tests

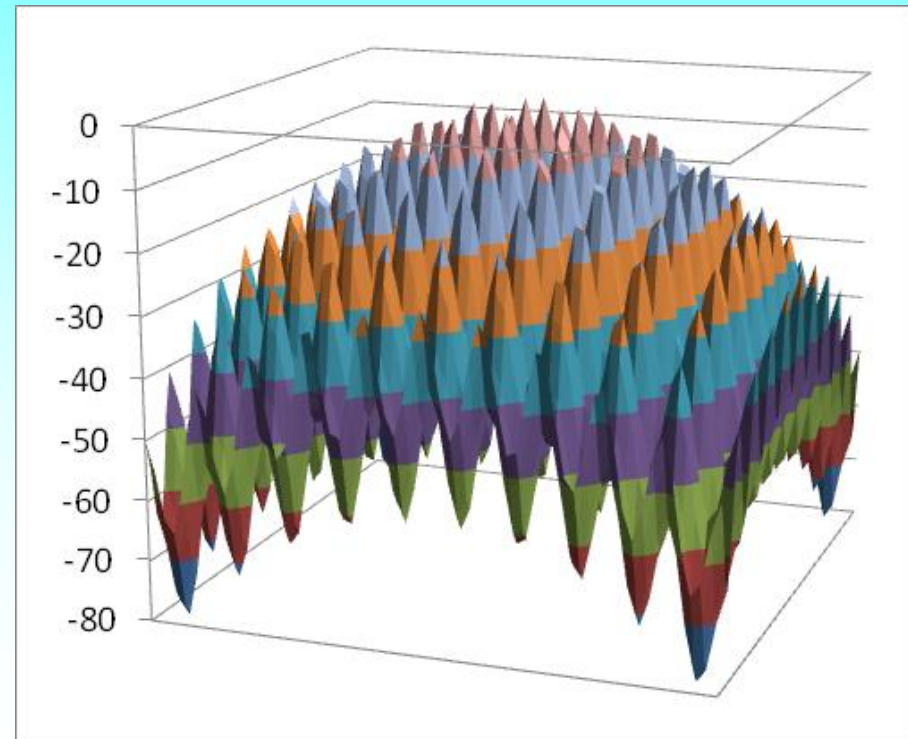
Rastrigin test

$$f(x) = nA + \sum_{i=1}^n (x_i^2 - A \cos(2\pi x_i))$$

where $x_i \in [-5.12, 5.12]$

for $i=1, \dots, n$, $a=20$, $b=0.2$, $c=2\pi$

Maximum is $f(x_i=0) = 0$



Numerical tests

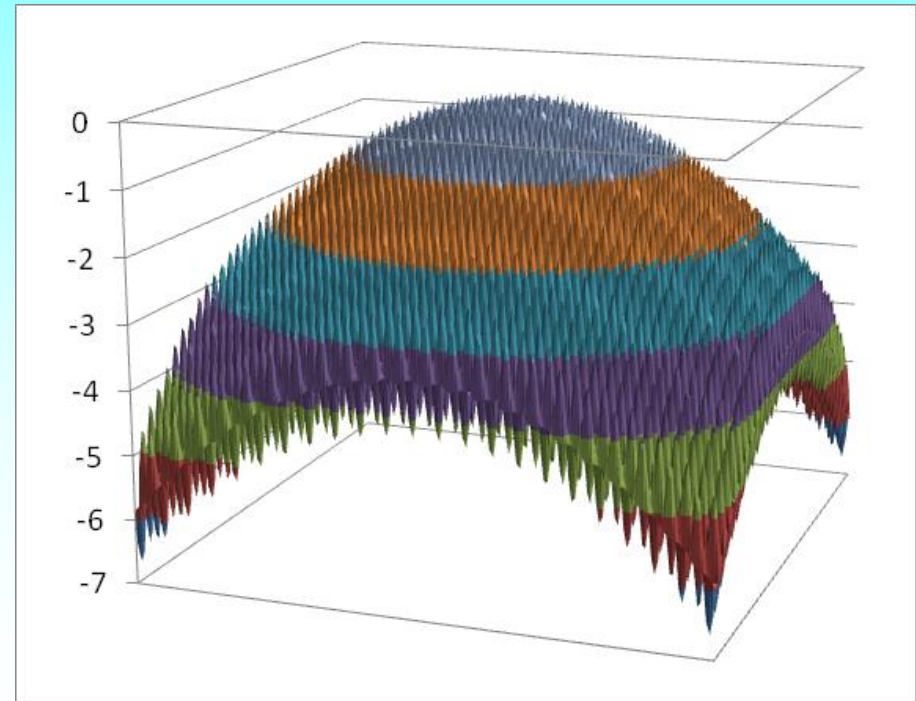
Griewank test

$$f(x_i) = \sum_{i=1}^n \frac{x_i^2}{4000} - \prod_{i=1}^n \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

where $x_i \in [-600, 600]$

for $i=1, \dots, n$, $a=20$, $b=0.2$, $c=2\pi$

Maximum is $f(x_i=0) = 0$



Numerical tests

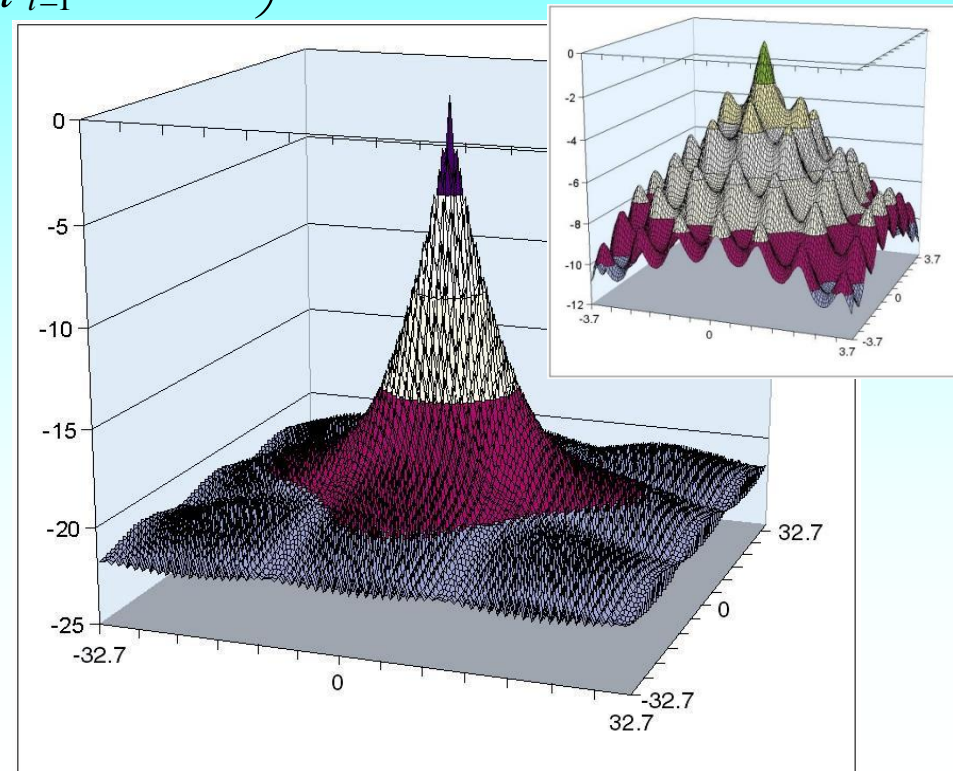
Ackley test

$$f(x) = a \exp \left[-b \left(\frac{1}{n} \sum_{i=1}^n x_i^2 \right) \right]^{1/2} + \exp \left(\frac{1}{n} \sum_{i=1}^n \cos(cx_i) \right) - a - \exp(1)$$

where $x_i \in [-32.0, 32.0]$

for $i=1, \dots, n$, $a=20$, $b=0.2$, $c=2\pi$

Maximum is $f(x_i=0) = 0$



Numerical tests

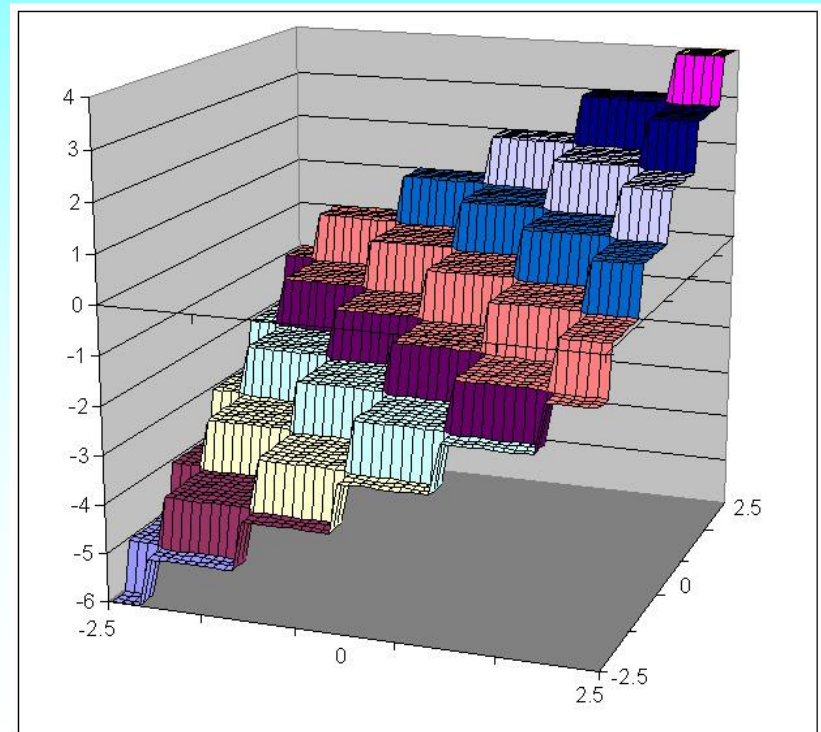
Step test

$$f(x_i) = \sum_{i=1}^n \lfloor x_i \rfloor$$

where $x_i \in [-2.5, 2.5]$
for $i=1, \dots, n-1$,

Maximum is dependant on dimensions
number.

Maximum is unknown.



Numerical tests

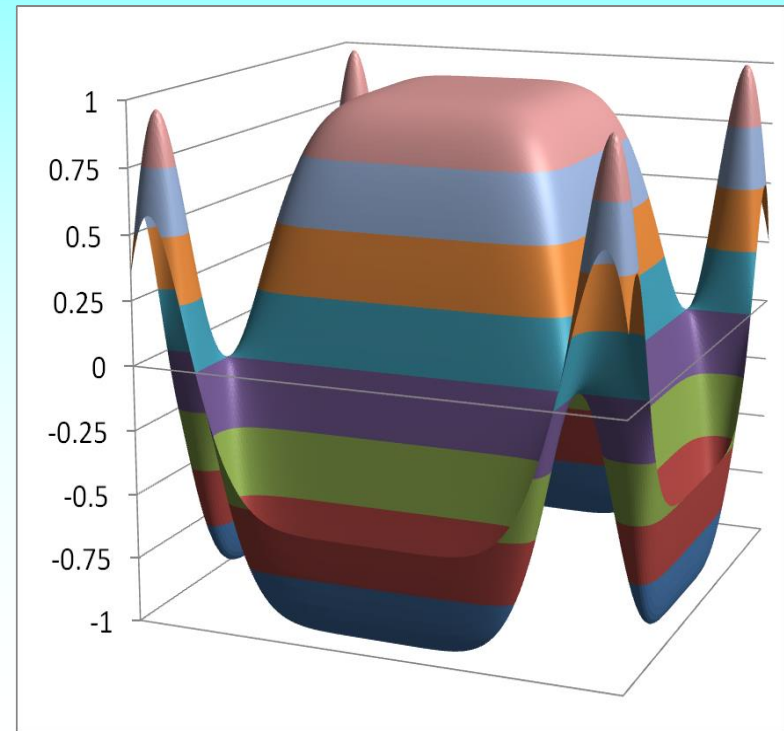
Norwegian test

$$\prod_{i=1}^n \left(\cos(\pi x_i^3) \left(\frac{99 + x_i}{100} \right) \right)$$

where $x_i \in [-1.1, 1.1]$ for $i=1, \dots, n$

Maximum is dependant on dimensions number.

Maximum is unknown.



Experimental settings

- **Computer system**
 - CPU Intel i7 3960x overclocked to 4600 MHz
 - CPU air cooler Noctua NH-D14 - modified
 - RAM - G.SkillTridentX 1866 MHz,
 - solid state disk - SanDisk Extreme SSD SATA III
 - motherboard ASUS Rampage VI



Experimental settings

- **Numerical tests - 2000 and 2016 dimensional**
 - Michalewicz test
 - Rosenbrock test
 - Rastrigin test
 - Griewank test
 - Step test
 - Ackley test
 - Norwegian test (2016 dimension only)
- **Experiments limits:**
 - 2 000 000 000 function evaluations
 - 20 000 000 000 function evaluations (Rosenbrock test only)

Experimental results

- Maximal results for 2000 dimensions
- Achieved by Free Search, May 2015

Test	Function evaluations	Maximal value
Michalewicz	$2 \cdot 10^9$	1999.48
Rosenbrock	$2 \cdot 10^{10}$	-0.00448202
Rastrigin	$2 \cdot 10^9$	-0.00000328
Griewank	$2 \cdot 10^9$	-0.02460318
Step	$2 \cdot 10^9$	4000.0000
Ackley	$2 \cdot 10^9$	-0.00020573
Norwegian	$2 \cdot 10^9$	n/a

Experimental results

- Maximal results for 2016 dimensions
- Achieved by Free Search, September 2015

Test	Function evaluations	Maximal value
Michalewicz	$2 \cdot 10^9$	2015.48
Rosenbrock	$2 \cdot 10^{10}$	-0.0034203
Rastrigin	$2 \cdot 10^9$	-0.00000319
Griewank	$2 \cdot 10^9$	-0.00000058
Step	$2 \cdot 10^9$	4032.0000
Ackley	$2 \cdot 10^9$	-0.0006701
Norwegian	$2 \cdot 10^9$	0.63724

Experimental results

- Maximal results for 2000 and 2016 dimensions
- Achieved for $2 \cdot 10^9$ Function evaluations

Ackley	2000	2016
Maximal	-0.0002057	-0.0006701
Average	-0.0003373	-0.0048272
Standard Deviation	0.000101977	0.002216941

Experimental results

- Maximal results for 2000 and 2016 dimensions
- Achieved for $2 \cdot 10^9$ Function evaluations

Step	2000	2016
Maximal	4000.0000000	4032.0000000
Average	4000.0000000	4032.0000000
Standard Deviation	0	0

Experimental results - time for calculations

- Time in minutes for one experiment for 2016 dimensions

Test	Function evaluations	Time minimal
Michalewicz	$2 \cdot 10^9$	5615
Rosenbrock	$2 \cdot 10^{10}$	6398
Rastrigin	$2 \cdot 10^9$	1383
Griewank	$2 \cdot 10^9$	1916
Ackley	$2 \cdot 10^9$	1376
Norwegian	$2 \cdot 10^9$	1452

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - time deviation

- Time in minutes for one experiment limited to $2 \cdot 10^9$

Test	Time in minutes
Michalewicz	5623
	5620
	5632
	5615

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - time deviation

- Time in minutes for one experiment limited to $2 \cdot 10^9$

Test	Time in minutes
Rastrigin	1383.5
	1384
	1384.83
	1385.6

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - time deviation

- Time in minutes for one experiment limited to $2 \cdot 10^9$

Test	Time in minutes
Griewank	1929
	1927
	1916
	1919

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - time deviation

- Time in minutes for one experiment limited to $2 \cdot 10^9$

Test	Time in minutes
Ackley	1376
	1377.83
	1380.6

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - time deviation

- Time in minutes for one experiment limited to $2 \cdot 10^{10}$

Test	Time in minutes
Rosenbrock	6460
	6456
	6434
	6398

- CPU 4.6 GHz
- RAM 1866 MHz

Experimental results - energy usage

- Energy and cost for $2 \cdot 10^9$ FEs for 2016 dimensional test

Test	Time minimal	Energy CPU/PSU kWh		Cost CPU/PSU 16.02p per kWh	
Michalewicz	5615	2.65	10.13	£0.42	£1.62
Rosenbrock*	6398	3.02	11.54	£0.48	£1.85
Rastrigin	1383	0.65	2.49	£0.10	£0.40
Griewank	1916	0.90	3.44	£0.14	£0.55
Ackley	1376	0.64	2.45	£0.10	£0.39
Norwegian	1452	0.69	2.64	£0.11	£0.42

- CPU i7-3960X - TDP - 130W
- CPU overclocked to 4.6 GHz - TDP - 170W for 6 cores
- TDP per overclocked core 28.33W
- PSU - 650W max, estimated per core 108.33

* Rosenbrock test is for $2 \cdot 10^{10}$ FEs

Issues & Limitations

- Hardware issues & limitations
 - CPU speed
 - RAM speed
 - Other factors - buss speed, multicore, multiprocessing, multitasking
 - Cooling systems
 - Energy consumption & cost

Issues & Limitations

- Software issues & limitations
 - Search method abilities for solution identification
 - Search method abilities for solution clarification
 - Search method speed
 - Search method effectiveness

Further directions

- Hardware aspects
 - Enhancing Computer systems performance
 - Advanced cooling systems
 - Hardware heat reutilisation
- Software aspects
 - Higher dimensions tests evaluation
 - Exploration of other tests
 - Search methods improvement

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References

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Thank you