Free Search Applied to Large Constraint Optimisation Problem

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Abstract. The article presents experimental results achieved by Free Search on optimisation of 100 dimensional version of so called bump test problem. Free Search is adaptive heuristic algorithm. It operates on a set of solutions called population and it can be classified as population-based method. It gradually modifies a set of solutions according to the prior defined objective function. The aim of the study is to identify how Free Search can diverge from one starting location in the middle of the search space in comparison to start from random locations in the middle of the search space and start from stochastic locations uniformly generated within the whole search space. The results achieved form the experiments with above initialisation strategies are presented. A discussion focuses on the ability of Free Search to diverge from one location if the process stagnates in local trap during the search. The article presents, also, the values of the variables for the best achieved results, which could be used for comparison to other methods and further investigation.

1 Introduction

In this study Free Search [10, 12, 13] is applied to 100 dimensional variant of so-called bump problem [6, 7]. The test - is hard constraint non-linear optimisation problem generalised for multidimensional search space. It is widely discussed in the literature. Large research efforts have been directed towards its exploration. The optimum is unknown, and the best-achieved results are published [2, 5–9, 11, 13, 15–18]. "Its a difficult problem that has been studied in the scientific literature and no traditional optimization method has given a satisfactory result. The function is non-linear and the global maximum is unknown." [9] An earlier investigation of the bump problem, published in the literature applies Genetic Algorithm [6] and some evolutionary algorithms [2, 7] to twenty and fifty dimensional variants of the problem. These investigations accept 20000 and 50000 iterations limit respectively for 20 and 50 dimensions. Better results, published in the literature, for 20 and 50 dimensional variants are achieved by evolutionary algorithms modified for precise exploration of the space near to the constraint boundaries [8, 9, 18]. The best results, published in the literature [5], for the bump problem from 2 up to 50 dimensions achieved by asynchronous parallel evolutionary algorithm APEMA on distributed MIMD computational system indicates best value 0.80361910 for 20 dimensions and 0.83523753 for 50 dimensions. The best results achieved by Free Search are for 20 dimensions 0.80361910412558 and for 50 dimensions 0.83526234835811175. This investigation has no available publications for other algorithms for 100 dimensional variant of the bum problem. The search space of this test is continuous. It has many peaks. An essential condition of this test is start from single location in the middle of the search space. This condition guarantees start from location, which is relatively far from the maximal hill. In contrast to start from multiple start locations uniformly distributed within the search space, it eliminates possibility for starting from initial locations accidentally generated near to the best value. Start from one location facilitates a measurement of the divergence across the whole space and then convergence to the best value. The work presented in this article is a continuation of the experiments on this problem published earlier [11, 17]. The experiments with the bump optimisation problem for $n = 50$, $x_i \in (0, 10)$, $i = 1, \ldots, 50$ require, for clarification of the results with precision of seven decimal digits, exploration of 10^{400} solutions ... Free Search achieves the maximum with such precision after exploration of less than 10^9 solutions... The relation between the number of all possible values with a certain level of precision and the number of explored locations deserves attention. For 50-dimensional space the relation is $10^9/10^{400} \sim 1/10^{44}$. These results can be considered from two points of view. The first point of view is when these results are accepted as accidental. The algorithm is "lucky" to "guess" the results. If that point of view is accepted, it follows that the algorithm "guesses" the appropriate solution from 10^{44} possible for 50-dimensional space. ...Another point of view is to accept the results as an outcome of the intelligent behaviour modeled by the algorithm. Free Search abstracts from the search space essential knowledge. That knowledge leads to the particular behaviour and adaptation to the problem. In that case the relation between explored and all possible locations can be a quantitative measure of the level of abstraction. The second point of view that the algorithm models intelligent behaviour is accepted. Abstraction knowledge from the explored data space; learning, implemented as an improvement of the sensibility; and then individual decision-making for action, implemented as selection of the area for next exploration; can be considered as a model of artificial thinking [11]. The experiments with 100 dimensional variant in this study are harder and Free Search confirms its excellent exploration abilities on large-scale optimisation problems. In the article and in the tables the following notation is accepted: n is the number of dimensions. i is dimensions indicator, $i = 1, \ldots n$. x_i are initial start locations. X_{max} and X_{min} are search space boundaries. x_{max} are variables of an achieved local maximum. F_{max100} is a maximal achieved value for the objective function. r_i is random value, $r_i \in (0, 1)$.

2 Test Problem

The objective function and the conditions are:

$$\text{Maximise:} f(x_i) = \left| \sum_{i=1}^{n} cos^4(x_i) - 2 \prod_{i=1}^{n} cos^2(x_i) \right| / \sqrt{\sum_{i=1}^{n} i x_i^2}$$

$$\text{subject to:} \prod_{i=1}^{n} x_i > 0.75 \qquad (1)$$

and

$$\sum_{i=1}^{n} x_i < 15\frac{n}{2} \qquad (2)$$

for $0 < x_i < 10$, and $i = 1, \dots n,$, starting from $x_i = 5$, $i = 1, \dots, n$, where x_i are the variables (expressed in radians) and n is the number of dimensions [7].

3 Methodology

Free Search is applied to the 100 dimensional variant of the bump problem as follow: The population size is 10 (ten) individuals for all experiments. Four series of 320 experiments with four different start conditions are made:

- start from one location in the middle of the search space $x_i = 5$, $i = 1, \dots, n$; (This is an original condition of the test, ignored from majority authors due to inability of other methods to diverge successfully from one location [8, 9, 18]
- start from random locations in the middle of the search space $x_i = 4 + 2 * r_i$, $r_i \in (0, 1)$;
- start from locations stochastically generated within the whole search space $x_i = (X_{max} - X_{min}) * r_i$, $r_i \in (0, 1)$;
- additional experiments with start from the best achieved location $x_i = X_{max}$ are made. The last result of these experiments is presented also in Table 1.

For the first three series maximal neighbour space per iteration is restricted to 5% of the whole search space and sensibility is enhanced to 99.999% from the maximal. For the fourth series, in order to distinguish very near locations with very similar high quality, maximal neighbour space per iteration is restricted to 0.0005% from the whole search space and sensibility is enhanced to 99.9999999% from the maximal.

4 Experimental Results

The maximal values achieved from 320 experiments for 100-dimensional search space for the four different start conditions are presented in Table 1. In Table 1: $x_i = 5$ start from one location; $xi = 4 + ri$ start from random locations in the middle of the search space; $xi = (X_{max} - X_{min}) * r_i$ start from random locations

Table 1. Maximal values achieved for n = 100 bump problem

n	start from	iterations	constraint (1) > 0.75	F_{max100}
100	$x_i = 5$	100000	0.750305761739250	0.838376006873
100	$x_i = 4 + r_i$	100000	0.7750016251034640	0.836919742344
100	$x_i = (X_{max} - X_{min}) * r_i$	100000	0.750125157302112	0.838987184912
100	$x_j = X_{max}$	100000	0.750000000039071	0.845685456012

uniformly distributed within the whole search space; $xi = X_{max}$ start from a local sub-optimum.

The best results from 320 experiments with start from currently achieved best local sub-optimum $x_i = X_{max}$ are presented in Table 2. The variables' values for the best achieved objective function value for $n = 100$ are presented respectively in Table 3. The constraint parameter values are an indicator whether the found maximum belongs to the feasible region. They indicate also expected possible improvement and can be valuable for further research.

The best results presented on the Table 2 suggest that for 100-dimensional search space n = 100 with the precision of four decimal digits (0.0001) the optimum is $F_{opt100} = 0.8456$. Considering constraint value with high probability could be accepted that the optimal value for 100-dimensional variant of the bump problem is between 0.845685 and 0.845686. Clarification of this result could be a subject of future research. From previous experiments Free Search achieves:"For $n = 50$, $x_i \in (0, 10)$, $i = 1, \ldots, 50$ there are 10^{400} solutions with a precision of seven decimal digits. Free Search achieves the maximum with such precision after exploration of less than 10^9 solutions" [11].

For $n = 100$, $x_i \in (0, 10)$, $i = 1, \ldots, 100$ with a precision of three decimal digits there are 10^{400} solutions. To reach the result with this precision current version of Free Search needs exploration of more than 10^{12} solutions. These results suggest that perhaps 100-dimensional space is more complex than 50-dimensional space with the same size.

Let us note that: (1) these results are achieved on a probabilistic principle;(2) the search space is continuous and the results can be clarified to an arbitrary precision; (3) precision could be restricted from the hardware platform but not from the algorithm [15, 16] .

5 Discussion

Comparison between the best achieved result an the results achieved within 100000 iterations with start from (1) single location in the middle of the search space, (2) random locations in the middle of the search space and (3) random locations stochastically distributed within the whole search space suggests difference of around 1%, - respectively - 0.80% for start from single location; - 1.04%

Table 2. Best achieved results from 320 experiments start from $x_i = X_{max}$

$F_{max} = 0.84568545600962819$	$px = 0.7500000000777115$
$F_{max} = 0.8456854560029361$	$px = 0.75000000002732459$
$F_{max} = 0.84568545601228962$	$px = 0.75000000003907141$
$F_{max} = 0.84568545600160894$	$px = 0.75000000003325862$
$F_{max} = 0.84568545601179412$	$px = 0.7500000000164837$
$F_{max} = 0.84568545600045464$	$px = 0.7500000000902789$
$F_{max} = 0.84568545600126788$	$px = 0.75000000002984646$
$F_{max} = 0.84568545600471334$	$px = 0.75000000013973589$
$F_{max} = 0.84568545600142975$	$px = 0.75000000000654199$
$F_{max} = 0.84568545600266842$	$px = 0.75000000002129508$
$F_{max} = 0.84568545600390022$	$px = 0.75000000011346923$
$F_{max} = 0.84568545600150213$	$px = 0.75000000021696134$
$F_{max} = 0.84568545600245093$	$px = 0.75000000004234102$
$F_{max} = 0.84568545600018008$	$px = 0.75000000002819023$
$F_{max} = 0.84568545600770795$	$px = 0.75000000006311085$
$F_{max} = 0.84568545600403033$	$px = 0.75000000001039713$
$F_{max} = 0.84568545600069034$	$px = 0.75000000001963218$

for start from random locations in the middle of the search space; - and 0.87% for start from uniformly distributed locations. Consequently Free Search can diverge successfully starting from one location and then can reach the optimal hill with the same speed as start from random locations. This in high extent is a confirmation of the independence of the algorithm from the initial population published earlier [12, 15, 16]. Theoretically these results can be interpreted as an ability of the algorithm to abstract knowledge during the process of search and to utilise this knowledge for self-improvement and successful, satisfactory completion of the search process. Rational value of the results is a confirmation of the high adaptivity of Free Search, which can support scientific search and engineering design in large complex tasks. Ability to continue the search process starting from the best achieved from previous experiments location brings additional value to the method. By means of the engineering design practice Free Search can overcome stagnation and can continue the search process until reaching an acceptable value and an arbitrary precision.

Table 3. Variables values for the best achieved Fmax100=0.84568545601228962

x[0]=9.4220107126347745	x[1]=6.2826322016103955	x[2]=6.2683831748189975
x[3]=3.1685544118834987	x[4]=3.1614825164328604	x[5]=3.1544574053305716
x[6]=3.1474495832290019	x[7]=3.1404950479045355	x[8]=3.1335624853236599
x[9]=3.1266689747394079	x[10]=3.1197805974560233	x[11]=3.1129253611190442
x[12]=3.1061003015872641	x[13]=3.0992700218084379	x[14]=3.0924554966175832
x[15]=3.0856546345558589	x[16]=3.0788589079539115	x[17]=3.0720627263662514
x[18]=3.0652657000077048	x[19]=3.0584746707061194	x[20]=3.0516637034865339
x[21]=3.0448584949106863	x[22]=3.0380286236964169	x[23]=3.0311906892219178
x[24]=3.024325642570814	x[25]=3.0174564447484493	x[26]=3.0105540636060151
x[27]=3.0036206348205221	x[28]=2.996654984477289	x[29]=2.9896634779600619
x[30]=2.9612192303388043	x[31]=2.9755508892857203	x[32]=2.9684029022824512
x[33]=2.9612192303388043	x[34]=2.9539969359393266	x[35]=2.9467002904105652
x[36]=2.9393411868452524	x[37]=2.9319091645017501	x[38]=2.9243754870396326
x[39]=2.916779048836506	x[40]=0.48215961508911009	x[41]=0.48103824318067195
x[42]=0.47987816774664849	x[43]=0.47878167955313988	x[44]=0.47768451249416577
x[45]=0.47661282330477983	x[46]=0.47553403486022883	x[47]=0.47446785125492774
x[48]=0.47342756022045934	x[49]=0.47239007924579712	x[50]=0.47137147513069294
x[51]=0.47032878010013335	x[52]=0.46930402745591204	x[53]=0.46831098721684394
x[54]=0.46735104878920491	x[55]=0.46636407600760849	x[56]=0.46539891729486565
x[57]=0.46441864961851576	x[58]=0.46347276946009547	x[59]=0.46251323773794134
x[60]=0.4616002801440135	x[61]=0.46065790486354485	x[62]=0.45975509243021634
x[63]=0.45882641352502623	x[64]=0.45794463889695297	x[65]=0.45703335381561577
x[66]=0.45616083797292917	x[67]=0.45530545866090766	x[68]=0.4544181045335004
x[69]=0.45356136754388732	x[70]=0.45270186592373279	x[71]=0.45185207109498432
x[72]=0.45101805630688263	x[73]=0.45019912098968307	x[74]=0.44936498811072595
x[75]=0.44854068486603355	x[76]=0.44772742155880246	x[77]=0.44690550854857802
x[78]=0.44610180301058927	x[79]=0.44532932347961096	x[80]=0.44452998407145683
x[81]=0.4437461852789148	x[82]=0.44294433416028023	x[83]=0.44217581932603778
x[84]=0.44144485059511346	x[85]=0.44065197401737571	x[86]=0.43992044125167573
x[87]=0.43915615308675215	x[88]=0.43841742447730969	x[89]=0.43766920812505228
x[90]=0.43695640149660347	x[91]=0.43620589323988396	x[92]=0.43550809702824705
x[93]=0.43477896755895717	x[94]=0.43406590088092134	x[95]=0.43335315514811895
x[96]=0.43265970793420877	x[97]=0.43194897101473584	x[98]=0.43126789176092778
x[99]=0.43057416262765469		

Another aspect of the results is what is the overall computational cost for exploration of this multidimensional task. A product between the number of iterations and the number of individuals in the algorithm population could be considered as a good quantitative measure for overall computational cost. For all experiments population size is 10 (then) individuals. In comparison to other publications [7–9] this is low number of individuals, which lead to low computational cost. To guarantee diversification and high probability of variation of the dimensions values within the population other methods operate on higher number of individuals, pay high computational cost and require large or distributed computational systems and extensive redundant calculations [5, 18]. For his task Free Search minimises required computational resources to a single processor PC.

6 Conclusion

In summary Free Search demonstrates good exploration abilities on 100-dimensional variant of he bump problem. Implemented novel concepts lead to an excellent performance. The results suggest that: (1) FS is highly independent from the initial population; (2) the individuals in FS adapt effectively their behaviour during the optimisation process taking into account the constraints on the search space, (3) FS requires low computational resources and pays low computational cost keeping better exploration and search abilities than the methods tested with the bump problem and discussed in the literature [2, 5, 7, 9, 18] (4) FS can be reliable in solving real-world non-linear constraint optimisation problems. The results achieved on the bump optimisation problem illustrate the ability of Free Search for unlimited exploration. With Free Search, clarification of the desired results can continue until reaching an acceptable level of precision. Therefore, Free Search can contribute to the investigation of continuous, large (or hypothetically infinite), constrained search tasks. A capability for orientation and operation within multidimensional search space can contribute also to the studying of multidimensional spaces and to a better understanding of real space. Presented experimental results can be valuable for evaluation of other methods. The algorithm is a contribution to the research efforts in the domain of population-based search methods, and can contribute, also, in general to the Computer Science in exploration and investigation of large search and optimisation problems.

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