Free Search and Particle Swarm Optimisation applied to Non-constrained Test

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Abstract: This article presents an evaluation of Particle Swarm Optimisation (PSO) with variable inertia weight and Free Search (FS) with variable neighbour space applied to non-constrained numerical test. The objectives are to assess how high convergence speed reflects on adaptation to various test problems and to identify possible balance between convergence speed and adaptation, which allows the algorithms to complete successfully the process of search on heterogeneous tasks with limited computational resources within a reasonable finite time and with acceptable for engineering purposes precision. Modification strategies of both algorithms are compared in terms of their ability for search space exploration. Five numerical tests are explored. Achieved experimental results are presented and analysed.

Keywords: Optimisation, Free Search, Particle Swarm Optimisation, Convergence Speed, Adaptive Methods.

1. Introduction

Investigation on tuning and improvement of convergence speed on various optimisation and search methods attracts research efforts [1][5][8][15][18]. However according to other publications high convergence speed usually increases probability for trapping in suboptimal solutions [7][13][17]. In order to identify a balance between high convergence speed and low probability for trapping this study focuses on Particle Swarm Optimization (PSO) and Free Search (FS). Low probability for trapping usually refers to the algorithms abilities for adaptation. When an algorithm can adapt to various task without changes of its search parameters is could be determined as adaptive algorithm [13]. Recent work investigates how different algorithms operate over different test problems in a limited time and also represents the importance of comparison between different methods' performance. Their execution may vary greatly whether they are applied to hard test problems [10]. For reducing the risk of uncertainties precise tests and analyses are made in terms of their ability for search space exploration [10].

Critical element for balanced search and good capabilities for any adaptation is modification strategy. For some algorithms modification strategy implicitly determines restrictions, which lead to low abilities for adaptation. Such algorithms require tuning of optimisation parameters for each particular task and if the method is applied to other task with the same parameters settings it cannot achieve acceptable results.

This study analyses modification strategy of PSO and FS and points how these strategies could be used in support of balanced set of search parameters. Then PSO and FS are applied to five well-known from the literature numerical optimisation problems generalised for multidimensional optimisation [6][13][18]. For fear assessment of the results each test is modified to ten dimensional variant.

1.1. Particle Swarm Optimization (PSO)

PSO is a classical algorithm used for search and optimisation [10]. Various modifications are published [9]. According to some publications PSO intends to model a social behaviour of a group of individuals whether it searches gradually for the optimum changing the values of the set of solutions [13]. However observation of process of search generated by PSO [18] suggests that its behaviour could be like self-organised particles in cloud systems.

Each particle (individual) shows a single intersection of all search dimensions and is defined as a potential solution to a test problem in multi-dimensional space. The particles appraise their position relative to an objective function (fitness) at every iteration whether particles in a local neighbourhood allocate memories of their best positions then use those memories to accommodate their own velocities, and thus positions [13]. Original concept is modified by adding inertia factor for velocities tuning [3][4]. This study uses modified PSO with variable inertia factor proposed earlier [13].

The velocity v is used to compute a new position for the particle as shown below:

$$x'_{id} = x_{id} + v_{id} \quad (1)$$

where x'_{id} is new position of particle i for dimension d, x_{id} is its current position and v_{id} is its velocity. The velocity vector v'_{id} for each particle is calculated using the best particles' achievement g_d, best for all population achievement P_{id} and inertia factor w according to the equation below:

$$v'_{id} = w^* v_{id} + n_1{}^*random(0,1)^*(P_{id} - x_{id}) + n_2{}^*random(0,1)^*(g_d - x_{id}) \quad (2)$$

Whether the constants n_1 (individual learning factor) and n_2 (social learning factor) are usually set with the equal values in terms of giving each component equal weight as the cognitive and social learning rate.

Both velocity component and inertia factor support adaptation to the explored test problem. PSO could be adjusted easily as it contains a few parameters only.

1.2. Free Search

Free Search could be best described as adaptive heuristic. This section refines the description published earlier [11], and aims to illustrate the manner in which a computational program can model processes that could be considered similar to thinking and reasoning. FS generates a new solution as deviation of a current one:

$$x = x_0 + \Delta x, \quad (3)$$

where x is a new solution, x_0 is a current solution and Δx is modification strategy. Other interpretation of Δx is that this is simply individuals' step. Individuals in FS explore the search space walking step by step. x, x_0 and Δx are vectors of real numbers. The modification strategy used in the algorithm is calculated according to the equation below:

$$\Delta x_{tji} = R_{ji} * (Xmax_i - Xmin_i) * random_{tji}(0,1), \quad (4)$$

where i indicates dimension; $i = 1,..,n$ for a multi-dimensional step; n is dimensions number; t is the current step $t = 1,..,T$. T is the step limit per walk; R_{ji} indicates the size of the idealised frame of the neighbourhood space for individual j within the dimension i. $random_{tji}(0,1)$ generates random values between 0 and 1. Δx_{tji} indicates the actual size of the step for step t of individual j within dimension i.

During the exploration an individual with a neighbourhood space, which exceeds search space boundaries, can perform global exploration whereas another individual with small neighbour space can make precise steps around one location.

The modification strategy is independent from the current or the best achievements and this is fundamental difference from PSO. The exploration performs heuristic trials based on stochastic divergence from one location. The concrete value of the neighbourhood space for a particular exploration defines the extent of uncertainty of the chosen individual. The exploration walk is followed by an individual assessment of the explored locations. The best location is marked with pheromone. The pheromone indicates the quality of the locations and may be considered as a result or cognition from previous activities. The assessment, during the exploration, is defined as follows:

$f_{tj} = f(x_{tji})$, $f_j = \max(f_{tj})$, (5)

where f_{tj} is the value of the objective function achieved from animal *j* for step *t*. f_j is the quality of the location marked with pheromone from an individual after one exploration. The pheromone generation is generalised for the whole population:

$P_j = f_j / \max(f_j)$, (6)

where $\max(f_j)$ is the best achieved value from the population for the exploration.

This is a normalisation of the explored problem to an idealised qualitative (or perhaps cognitive) space, in which the algorithm operates. This idealised space uses for a model an idealised space of notions in thought of biological systems, in which they generate decisions. The normalisation of any particular search space to one idealised space supports adaptation and successful performance across variety of problems without additional external adjustments. The sensibility generation is:

$S_j = Smin + \Delta S_j$, (7)

where

$\Delta S_j = (Smax – Smin)*random(0,1)$ (8)

Smin and *Smax* are minimal and maximal possible values of the sensibility.

Smin = Pmin, Smax = Pmax.

Pmin and *Pmax* are minimal and maximal possible values of the pheromone marks. The process continues with selection of a start location for a new exploratory walk. The ability for decision-making based on the achieved from the exploration (which can be in contradiction with the existing assumptions about the problem during the implementation of the algorithm) supports a good performance across variety of problems, adaptation and self-regulation without additional external adjustments. Selection for a start location x_{0j} for an exploration walk is:

$x_{0j} = x_k \ (P_k \geq S_j)$, (9)

where *j* = 1, ..., *m*, *j* is the number of the individuals; *k* = 1, ..., *m*, *k* is the number of the location marked with pheromone; x_{0j} is the start location selected from animal number *j*. After the exploration follows termination.

A specific original peculiarity of Free Search, which has no analogue in other evolutionary algorithms, is a variable called sense. It can be likened as a quantitative indicator of sensibility. The algorithm tunes the sensibility during the process of search as function of the explored problem. The same algorithm makes different regulations of the sense during the exploration of different problems. This is considered to be a model of adaptation [13]. The variable sense distinguishes the individuals from the solutions. The individuals are search agents differentiated from the explored solutions and detached from the problems' search space. A solution in FS is a location from a continuous space marked with pheromone. The individuals explore, select, evaluate and mark these solutions.

An individual in FS can be described by the abstraction – an entity, which can move and can evaluate (against particular criteria) locations from the search space thereby indicating their quality. The indicators can be interpreted as a record of previous activities. The individual can identify the pheromone marks from previous activities and can use them to decide where and how to move. It is assumed that all these characteristics are typical of the manner in which animals behave in nature. Therefore the individuals in FS are called animals. The variable sense when considered in conjunction with the pheromone marks can be interpreted as personal knowledge, which the individual uses to decide where to move. The variable sense plays the role of a tool for regulation of divergence and convergence within the search process and a tool for guiding the exploration [11].

2. Modification Strategies Comparison

Comparing modification strategies of PSO and FS it can be identified that inertia weight in PSO plays the same role as neighbour space in FS. Both of them are absolute values, which reflect on convergence speed, abilities for adaptation. Essential difference is that in PSO the current best achievements for the population and for the particles explicitly determine generation of new velocity and then the new individuals. The locations, within the area of the possible velocity values around an individual, have non-zero probability for access. However, for the rest of the search space the probability for access is zero. Increasing inertia factor increases the area with nonzero probability for access. This decreases probability for trapping however this critically decreases convergence speed. Decreasing inertia factor directly decreases the area with nonzero probability for access. This increases the convergence speed to the located peak. However, if this is a local peak PSO has no mechanism to escape. In FS mechanism for trapping avoidance is based on individuals' sense.

3. Numerical Tests

3.1. Griewank Test Function

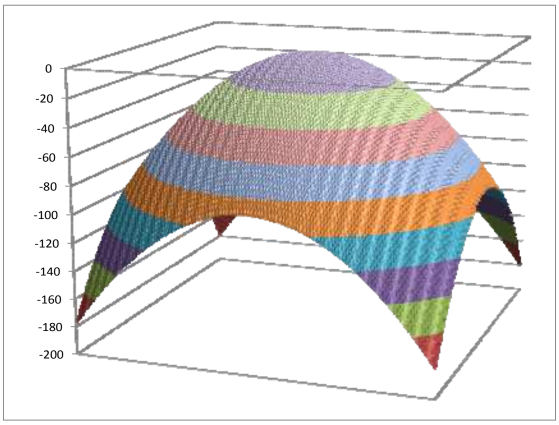

Figure 1: Griewank test

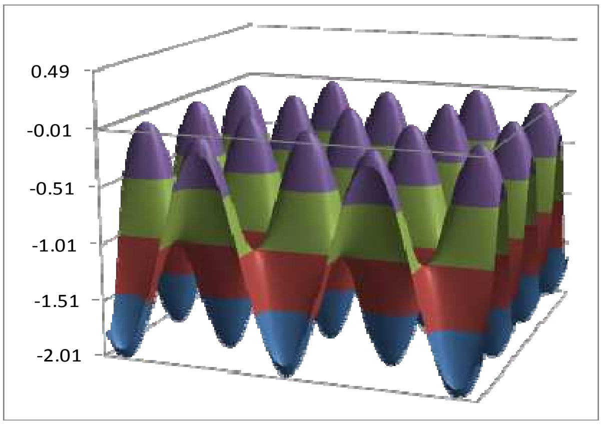

Figure 2: Griewank test – top of the hill

$$f\left(x_i\right)=1+\frac{1}{4000}\sum_{i=1}^{n}x_i^2-\prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) \qquad (10)$$

where $x_i \in$ [-600.0, 600.0]. Global solution is f(0.0, 0.0) = 0.0.

3.2. Norwegian Test Function

$$f\left(x_i\right)=\prod_{i=1}^{n}\left(\cos(\pi x_i^3)\left(\frac{99+x_i}{100}\right)\right) \qquad (11)$$

where $x_i \in$ [-1.1, 1.1]. Global solution is f(1.00011, 1.00011) = 1.00000113. All local peaks are below 0.99.

3.3. Rastrigin Test Function

$$f(x) = nA + \sum_{i=1}^{n} (x_i^2 - A\cos(2\pi x_i)) \quad (12)$$

where $x_i \in$ [-5.12, 5.12]. Global solution is *f*(0.0, 0.0) = 0.0.

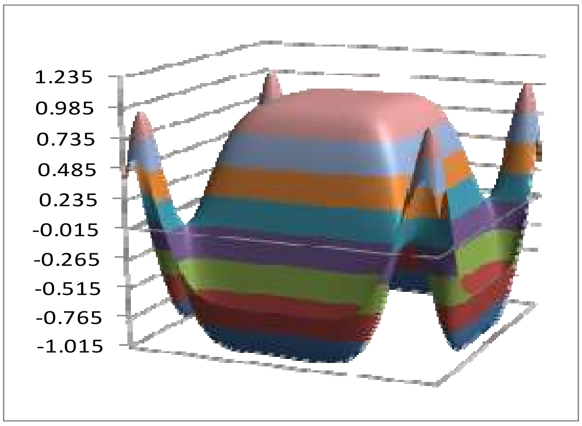

Figure 3: Norwegian test

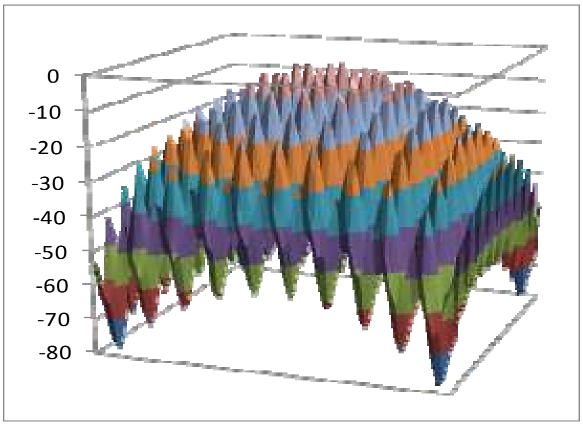

Figure 4: Rastrigin test

3.4. Rosenbrock Test Function

$$f(x_i) = -\sum_{i=1}^{n-1} [100 * (x_{i+1} - x_i^2)^2 + (x_i - 1)^2] \quad (13)$$

where $x_i \in$ [-500, 500]. Global solution is *f*(1.0, 1.0) = 0.0. The function is shown on Figure 5.

3.5. Sphere Test Function

$$f(x_i) = -\sum_{i=1}^{n} x_i^2 \quad (14)$$

where $x_i \in$ [-512, 512]. Global solution is *f*(0.0, 0.0) = 0.0.

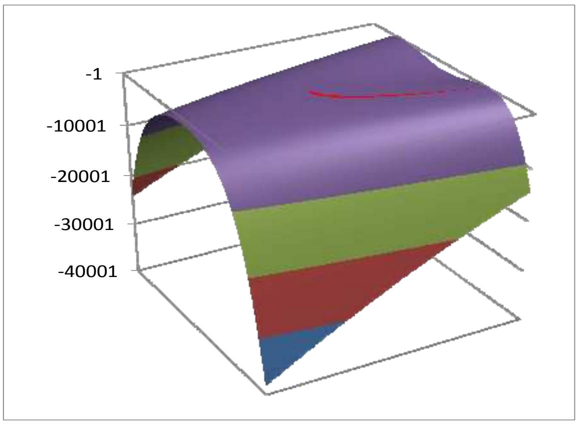

Figure 5: Rosenbrock test

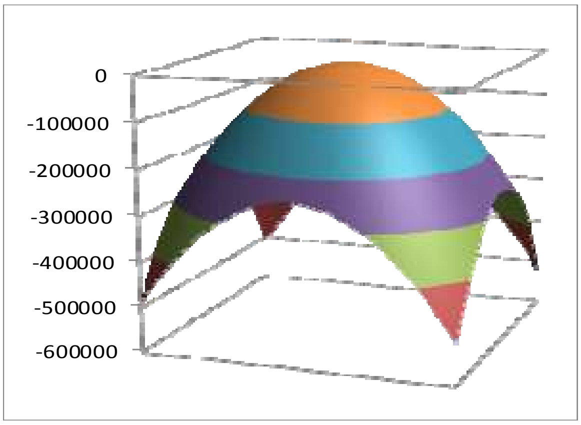

Figure 6: Sphere numerical test

4. Experimental Results

All tests are evaluated for 10 dimensions. For all experiments for both FS and PSO population is 10 individuals. PSO experiments are limited to 2000 and 20000 iterations per each test. Total number of test function evaluations for 10 individuals are accordingly: 2000x10 = 20000 and 20000x10 = 200000.

FS experiments are limited to 400 and 4000 explorations with 5 steps per exploration. Total number of test function evaluations for 10 individuals are:
400x5x10 = 20000 and 4000x5x10 = 200000.

For all test PSO is applied with the variable inertia factor which enhances to some extent its ability for adaptation. Individual learning factor is 2.0. Group learning factor is 2.0. Inertia weight varies within the interval 0.1 - 1.0 with step 0.1. Initialisation is stochastic:

$x_{0ji} = Xmin_i + (Xmax_i - Xmin_i)*random_{ji}(0, 1),$ (15)

where $Xmax_i - Xmin_i$ are search space boundaries; *j* is individual, *i* is dimension.

For all test FS is used with its standard set of parameters – population size – 10; steps per exploration 5. In order to provide equal conditions for testing with PSO neighbour space varies within the interval 0.1 - 1.0 with step 0.1. Sensibility randomly varies within the interval 0.99999 - 1.0.

To reduce probability for dependence on initialisation, 32 experiments with different initialisation per each inertia value are completed for both FS and PSO. This corresponds to 320 experiments per test per method in total. FS is evaluated additionally to the same number of experiments but with start for all individuals from a single location purposefully selected away from the global optimum. The single location for all tests is defined as:

$x_{0ji} = Xmin_i + 0.1*(Xmax_i - Xmin_i),$ (16)

where $Xmax_i - Xmin_i$ are search space boundaries; *j* is individual; *i* is dimension.

Achieved best values from 320 experiments are presented in the tables below.

Table 1: PSO start from random locations

Function	Iterations	
	2000	20000
Rastrigin	-4.9748	-2.98488
Griewank	-0.009857	-0.007396
Rosenbrock	-0.00115796	-0.0023069
Norwegian	0.950406	0.951045
Sphere	-2.9557E-078	0.000000

Table 2: FS start from random locations

Function	Iterations	
	2000	20000
Rastrigin	-0.000129	-1.4571E-06
Griewank	-0.002767	-1.3650E-06
Rosenbrock	-8.4371E-05	-1.5643E-05
Norwegian	0.966949	0.99998
Sphere	-0.004359	-0.000109

Table 3: FS start from single location		
Function	Iterations	
	2000	20000
Rastrigin	-0.000281	-1.0618E-06
Griewank	-0.000699	-3.08E-06
Rosenbrock	-2.3184E-05	-2.0315E-06
Norwegian	0.980593	0.999974
Sphere	-0.010623	-4.6E-05

PSO demonstrates unapproachable convergence on 10 dimensional Sphere test. It reaches the optimum for less than 2000 iterations. On Griewank, Rosenbrock tests PSO shows dependence on initialisation and accidental events. If initialisation is appropriate it achieves optimal values for ten dimensional variant of these tests within 2000 iterations with precision below 0.01%. Although on Rosenbrock tests from 320 experiments limited to 20000 iterations PSO reaches more results close to the optimum its best result was not better than for 2000 iterations. Identification of reasons for this requires further investigation.

PSO cannot escape from local trap of Norwegian test within 20000 iterations and has difficulties on Rastrigin test. In order to resolve these tests PSO needs retuning of its parameters.

Standard FS configuration has medium convergence speed. iFS [18] is the Free Search modification with highest convergence speed published in the literature. Within 400 explorations (corresponding to 2000 iterations) FS cannot escape from trapping for Norwegian test and cannot reach close to zero value on Rosenbrock test. However for 4000 explorations (corresponding to 20000 iterations) FS resolves all tests for both start from random locations and start from single location. For experiments with start from single location FS confirm its abilities to diverge over the search space and then to identify optimum.

5. Conclusion

Presented study contributes to the knowledge in adaptive computing and heuristic methods in terms of identification of balance between convergence speed and abilities for adaptation for PSO and FS applied to non-constrained global optimisation.

Experimental results confirms: (1) published earlier evaluation that PSO [13] has excellent convergence in local search but is not adaptive enough in global optimization due to constraints produced from the best particles' and all population achievements' P_i and g; (2) standard FS configuration has balance between convergence (which is medium) and abilities for adaptation and can resolve various problems without retuning of its parameters within reasonable amount of time and with acceptable for engineering purposes precision.

Acknowledgement
Preparation of this paper is supported by Southampton Solent University, Research and Enterprise Fund, Grant 516/17062011.

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