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Reducing railway-induced ground-borne vibration by using open trenches and soft-filled barriers

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ABSTRACT

A trench can act as a barrier to ground vibration and is a potential mitigation measure for low frequency vibration induced by surface railways. However, to be effective at very low frequencies the depth required becomes impractical. Nevertheless, for soil with a layered structure in the top few metres, if a trench can be arranged to cut through the upper, soft layer of soil, it can be effective in reducing the most important components of vibration from the trains. This study considers the possibility of using such a realistically feasible solution. Barriers containing a soft fill material are also considered. The study uses coupled finite element / boundary element models expressed in terms of the axial wavenumber. It is found to be important to include the track in the model as this determines how the load is distributed at the soil's surface which significantly affects the insertion loss of the barrier. Calculations are presented for a range of typical layered grounds in which the depth of the upper soil layer is varied. Variations in the width and depth of the trench or barrier are also considered. The results show that, in all ground conditions considered, the notional rectangular open trench performs best. The depth is the most important parameter whereas the width has only a small influence on its performance. More practical arrangements are also considered in which the sides of the trench are angled or a retaining structure is used. Barriers consisting of a soft fill material are shown to be much less effective than an open trench but still have some potential benefit. It is found that the stiffness of the barrier material and not its impedance is the most important material parameter.

1. INTRODUCTION

Ground vibration from trains is an increasingly important environmental issue. It manifests itself in two ways: low frequency vibration in the range 2-80 Hz is perceived by lineside residents as whole-body feelable vibration, whereas higher frequency vibration in the range 16-250 Hz is radiated as sound inside buildings and is known as ground-borne noise [1, 2]. Trains running on surface railways, particularly where the ground is soft, often produce vibration with its highest components in the range below 40 Hz, which is mainly experienced as feelable vibration. Velocity amplitudes are typically between 0.1 and 1 mm/s. Conversely, trains running in tunnels tend to produce higher frequency vibration at considerably lower amplitudes for which ground-borne noise is more important.

Ground can often be represented as a series of parallel soil layers [1, 2]. Where a shallow surface layer of softer soil overlies stiffer soil layers, the vibration is characterised by the onset of high vibration levels above a certain frequency that depends on the layer depth and wavespeeds in this upper soil layer. In many practical cases the upper layer has a thickness of 2-6 m and the corresponding cut-on frequency is typically in the range 10-30 Hz. As a result the maximum vibration often occurs in the range between 10 and 40 Hz and this must be borne in mind when considering mitigation measures [3]. At other sites where the soft soil has a greater depth and a lower wavespeed, the maximum frequency may even be lower than 10 Hz.

In principle there are a number of possible ways to reduce railway-induced vibration [1, 2, 3], including changes to the vehicle [4], modifying the track [5] or the ground beneath it [6, 7] or introducing a barrier of some form beside the track. A stiff barrier in the ground beside the track, constructed for example from concrete, can give effective shielding of vibration [8-10]. It has been shown in [9] that bending waves in the stiff barrier are important. A row of heavy masses on the ground surface has also been shown to give attenuation of vibration at frequencies above the resonance frequency of the masses on the ground stiffness [11].

An open trench is commonly used to attenuate ground vibration from machinery [12]. This can act in a similar way to a noise barrier for airborne sound; vibration is diffracted underneath the barrier and only a fraction of the original vibration reaches the ‘shadow zone’ behind it. An ideal open trench with vertical sides is not stable so in practice it requires either sloping sides or reinforcing walls [13]. Alternatively, a trench may be filled with a soft material. This is generally less effective as vibration is transmitted through the fill material as well as being diffracted underneath the trench. The fill material should therefore be much softer than the surrounding soil while being capable of balancing the surrounding earth pressure [14]. Common fill materials considered include bentonite, soil-bentonite mixtures [15], expanded polystyrene (EPS) [16, 17] and other geo-foam materials such as polyurethane [18].

Open trenches have long been considered as a possible solution for ground vibration from machinery as well as railways. Early field tests were presented by Woods [12, 19]. The results were presented as the amplitude reduction ratios and a reduction of at least 0.25 (i.e. 12 dB) was considered ‘effective’. This was achieved with a trench of depth at least 0.6 times the wavelength of Rayleigh waves. The width was found not to be critical. Trenches in the far-field from the source were found to be less effective. More recently Alzawi and El Naggar [18] presented field measurements of open and soft-filled trenches. They confirmed the conclusions from Woods [12] for an open trench but found that for a trench filled with geo-foam the reductions for a depth (normalised to the Rayleigh wavelength) of 0.6 were reduced to the equivalent of about 8-10 dB. Celebi et al [20] also present some field measurements of a concrete-lined trench. Kim et al [21] describe an experiment with a trench filled with a mat made of rubber chips.

Massarsch [14] gave a review of the use of gas cushions. These were developed and patented by Franki International in the 1980s [22] and allow a soft-filled barrier with a very low stiffness to be used. A number of field installations to isolate buildings from railways were described. The depth varied between 6 and 12 m. He indicated, considering the transmission coefficient at the interface between two semi-infinite media, that the transmission coefficient through the barrier should depend on the ratio of the impedances of the barrier and the soil.

In [23] some trials of trenches beside both railways and tramways are reported. For example, measurements for a 3.5 m deep trench showed reductions of 10 dB above 16 Hz [23]. Yoshima [24] presented results for piled trenches adjacent to a high speed line. At one site the trench was 4 m deep and at another it was 10 m deep; the frequency-weighted vibration was reduced by around 10 dB. Lang [25] found that a 1.5 m deep trench filled with railway ballast reduced vibration from a tram line by around 10 dB above 31.5 Hz, although the benefit decreased at larger distances. François et al [26] described an installation of an 8 m deep screen consisting of polystyrene, concrete and bentonite alongside a tram track. The results obtained were disappointing, this being attributed to an insufficient stiffness ratio between the barrier and the surrounding soil. Although various results have been cited, published measurement results for track-side trenches are scarce and they are difficult to generalise. The vibration reduction will depend strongly on the ground conditions as well as on track and vehicle design. Numerical analysis can provide an alternative which allows a more systematic understanding to be developed.

The main approaches used to model soil-structure interaction [27] are the boundary element (BE) method and the finite element (FE) method. To prevent reflections at artificial boundaries of the model, the FE approach is often used together with the BE method or with infinite elements (IE) or other non-reflecting boundaries.

May and Bolt [28] used two-dimensional (2D) FE models to study the effect of an open trench on incident waves of different types. They confirmed that a non-dimensional depth of 0.6 is sufficient to obtain a reduction of 12 dB. Beskos et al [29, 30, 31] introduced 2D and 3D BE models of open trenches. Their results confirmed that a non-dimensional depth of at least 0.6 is required to give an amplitude reduction of 0.25 (12 dB). Ahmad and Al-Hussaini [32] also used 2D BE models to study an open trench, extended to 3D in [15]. Klein et al [33] used a 3D BE method to study an open trench and Kattis et al [34] studied a row of piles. Ekanayake et al [17] used an FE model to study an open trench and one filled with expanded polystyrene (EPS) geofoam or water.

Most published results are based on a homogeneous ground. May and Bolt [28] included a surface layer but the wavespeeds differed only by 20%. Leung et al [31, 35] considered a layered or continuously non-homogeneous ground. They found that, for a softer layer over a stiffer half-space, the effectiveness of a trench is significantly reduced compared with a homogeneous material. A depth of twice the Rayleigh wavelength was found to be necessary where the upper layer was shallower than 2.5 wavelengths [31]. Ahmad and Al-Hussaini [32] also gave results for an open trench in a layered half-space with similar conclusions.

The numerical modelling of trenches has been extended to study the effect on railway vibration by a number of authors. Yang and Hung [36] used 2D FE/IE models of an open trench and found that a high Poisson's ratio of the soil meant that an open trench had to be deeper for the same effectiveness. Hubert et al [37] used 3D BEM in the time domain to study a rigid track on a half-space and the introduction of an open trench. Results are given only for two example frequencies. Adam and von Estorff [38] used a 2D coupled FE/BE model to study the transmission of vibration from a railway to a nearby building. Both an open trench and a trench filled with soil-bentonite mixture were considered. A 3D FE/BE model which operates in the frame of reference moving with the load was presented by Andersen and Nielsen [39]. An open trench and a trench filled with rubber chips were considered as well as other options.

Connolly et al [40] used a 3D FE model to study the effects of an open trench adjacent to a railway line. It was shown that the depth is important but the width is not. Results were shown in terms of non-dimensional parameters expressed in terms of a single equivalent frequency representing the train pass-by loading. A 3D FE model was also used by Younesian and Sadri [41] to study trenches with different cross-sections.

Hung et al [42] used a 2.5D FE/IE approach to study an open trench and a concrete-filled trench. In such an approach, the mesh is two-dimensional and the third dimension is represented in the wavenumber domain. Barbosa et al [43] presented results from a full 2.5D BE/FE model of the track and ground including a moving excitation. Results were given for an open trench, a trench filled with geo-foam and a concrete barrier. They found that it is important to include the moving load on the track, especially for the stiff barrier.

From both measurements and computer modelling of railway vibration, it has been found that the most important frequency components are controlled by vibration propagation in upper layers of soil that are often only a few metres deep [1]. This suggests that a trench that cuts through such a surface layer may have the potential to give significant reductions of the most important parts of the vibration spectrum. Jones et al [13] used a two-dimensional boundary element model to study rectangular trenches in a layered ground, considering the effect of their depth and position. The ground consisted of a 2 m layer of alluvial soil over a substratum of stiffer material (say, gravel beds). This study was extended in [44] to include trenches with a retaining structure or a slope, again using two-dimensional modelling.

In the present paper, a study is presented of the potential of open trenches and soft-filled barriers to mitigate low frequency railway vibration, paying particular attention to the influence of ground layering. A three-dimensional analysis is carried out using the wavenumber (i.e. 2.5D) finite element-boundary element method [45-47]. As the layered ground properties can be important in determining the dominant frequency range of vibration, and potentially in the mechanism of attenuation due to a trench, various different layered grounds are considered. A particular issue that is investigated is whether it is sufficient to ensure the impedance ratio of the ground to the fill material is large enough, as used for example by Massarsch [14], or whether other properties of the barrier are more appropriate for this purpose.

The modelling approach and the ground properties considered are introduced in Section 2. Results are concentrated on a set of notional but realistic ground properties in which the depth of the soft surface layer is varied. Section 3 presents results for open trenches of various depths, widths and shapes. In Section 4 the effect of filling the trench with a soft barrier material is presented. Finally in Section 5, to show the range of effects that can be achieved under different practical conditions, results are presented for three actual sites considered in previous work [10, 14]. Throughout, the excitation consists of a ‘line load’ made up of a series of incoherent point loads on the track at the positions of the axles of a train. These are used to determine insertion losses for various trenches and soft barriers. A separate calculation of the vibration from a passing train is combined with some examples of the insertion losses to demonstrate the effect of the trench or soft barrier on the spectrum of train vibration.

2. MODELS

2.1 2.5D finite element / boundary element model

An overview of the problem considered is shown in Fig. 1. The soil may have one or more layers and a trench is introduced with a depth that may be sufficient to penetrate through these layers; also shallower trenches are considered. The trench may be open or filled with a soft material. It is located at a distance *d* from the centre of a railway track which is situated directly on the ground surface.

The geometry is essentially two-dimensional and invariant in the third dimension. Under such conditions, it is possible to use a wavenumber transform in the axial direction in combination with a two-dimensional finite element / boundary element model. The full three-dimensional solution can be recovered by an inverse Fourier transform over wavenumber. This so-called 2.5D approach has been used widely to study railway vibration, e.g. [45-47]; it is more efficient than a full three-dimensional approach. Similarly, 2.5D finite element / infinite element models have been used [42, 48, 49].

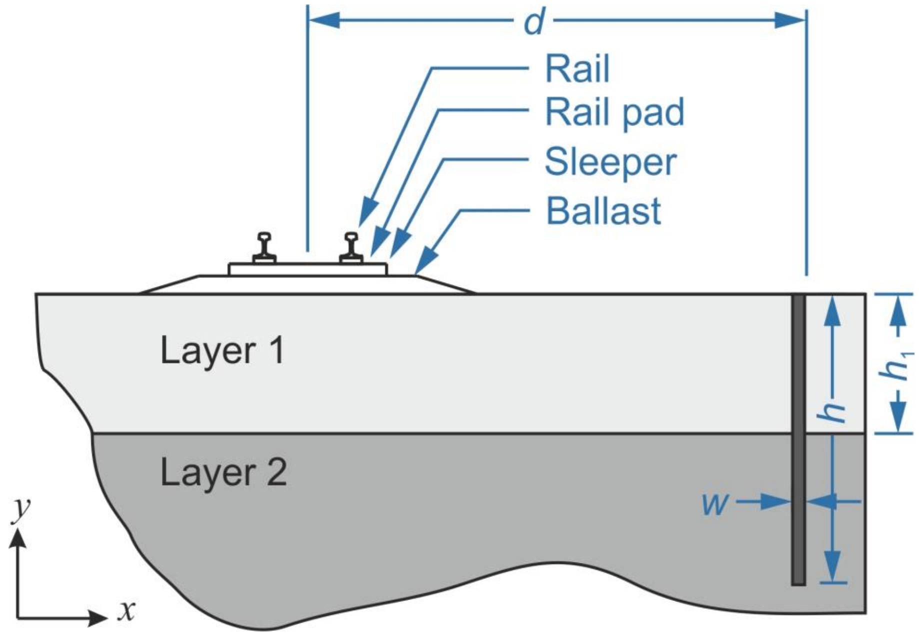

Fig. 1. Arrangement of a trench in a layered soil adjacent to a railway track showing the various parameters considered.

The 2.5D coupled FE-BE model used in the present study [50] uses boundary elements that are based on fundamental solutions of a homogeneous full space. Therefore the ground surface and any layer interfaces need to be carefully meshed to a sufficient distance. A special edge element is used to avoid the reflections at the end of the ground or layer mesh [45]. The track is represented using finite elements while the ground is modelled using boundary elements. Calculations have been made from 1.25 to 100 Hz with three calculation frequencies in each one-third octave band. Results are subsequently averaged into one-third octave bands for presentation. At each frequency 1024 wavenumbers are used covering a range of ±2 rad/m up to 5 Hz, ±4 rad/m up to 15 Hz and ±8 rad/m above this frequency.

A benchmark comparison has previously been made between the models of ISVR used here and a similar approach used by KU Leuven [47]. In the latter approach the BE model is based on 2.5D Green's functions of a horizontally layered half-space [51]. Results from this benchmark comparison showed good agreement between the two approaches [52].

2.2 Ground properties

In order to test the application of trenches and soft-filled barriers, a series of notional layered ground conditions are considered. The same ground parameters were considered in [7] to study the effect of subgrade stiffening. These properties are typical of sites investigated during the EU FP7 project RIVAS (Railway Induced Vibration Abatement Solutions) [53] and are based loosely on sites identified at Rubigen in Switzerland where an upper layer of silty sand with clay and gravel, with a thickness of 2–10 m, is located over a stiffer substratum of silty gravel with sand. The depth of the upper layer was found to vary considerably across the site. The soils are assumed to be unsaturated.

The material properties associated with these soil layers are listed in Table 1. The damping loss factor η is used to make the shear modulus and Young's modulus complex in the form $G^* = G(1+i\eta)$. The depth of the upper layer is varied, taking values $h_1 = 3$ and 6 m. In addition a case is considered in which the soil consists only of the softer soil (identified as $h_1 = \infty$) and another consisting only of the stiffer soil (identified as $h_1 = 0$ m). The ground surface is meshed with boundary elements having a node-to-node spacing of 0.25 m, giving at least 6 nodes per shear wavelength up to 100 Hz. The interface between the upper layer and the substratum is also meshed with boundary elements. The boundary elements extend to a distance of 34 m to one side of the load and 5 m to the other side.

Table 1. Soil characteristics adopted for the typical two-layer ground sites.

	Layer 1	Layer 2
Thickness [m]	0 / 3 / 6 / ∞	∞
Density [kg/m³]	2000	2000
Young’s modulus [GN/m^2]	0.120	1.915
Poisson’s ratio [-]	0.33	0.33
Shear wave speed [m/s]	150	600
Compressional wave speed [m/s]	298	1191
Damping loss factor [-]	0.03	0.1

Table 2. Characteristics of the track

Rail	Bending stiffness [MNm²]	6.4
	Mass per unit length [kg/m]	60
Rail pad	Stiffness [MN/m]	300
	Damping loss factor [-]	0.10
Sleeper	Length [m]	2.60
	Width [m]	0.25
	Height [m]	0.20
	Mass [kg]	325
	Young's modulus [GN/m²]	30
	Poisson's ratio [-]	0.15
	Sleeper spacing [m]	0.60
Ballast	Thickness [m]	0.30
	Shear wave velocity [m/s]	300
	Poisson's ratio [-]	1/3
	Density [kg/m³]	2000
	Damping loss factor [-]	0.04
	Upper width [m]	3.6
	Lower width [m]	5.6

In addition a railway track is added to the ground surface, represented by finite elements. Its parameters are given in Table 2. The sleepers and rail pads are replaced in the FE by equivalent continuous representations in which the density and Young's modulus are reduced from the values in Table 2 by a factor (0.25/0.6) where 0.25 m is the sleeper width and 0.6 m is the spacing between sleepers. To avoid introducing an incorrect bending stiffness along the track, the sleepers are modelled using orthotropic elements with the corresponding moduli set to very small values (setting them equal to zero leads to a singular constitutive matrix). The finite element mesh for the track and the boundary element mesh for part of the ground surface and layer interface are shown in Fig. 2. The track system, including rail pad, sleeper and ballast, is meshed with 4-node and 8-node quadrilateral finite elements. The rails, not visible in the figure, are represented by beam elements located on the upper surface of the rail pads.

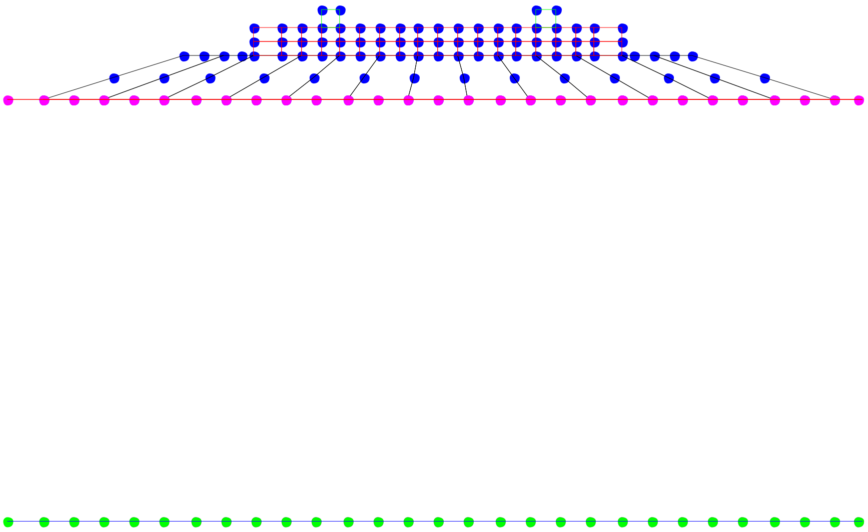

Fig. 2. Geometry of the 2.5D model for the railway track on a layered ground. Only part of the free surface and layer interface are shown.

2.3 Dispersion and response to unit loads

As discussed in [7], the characteristics of the layered soil have a large impact on the wave propagation. Dispersion plots for the soil with an upper layer of depth 3 m are reproduced in

Fig. 3 [7]. These were obtained using an analytical model of the layered soil and indicate the wavenumbers of free surface waves that can occur in the soil at each frequency. At low frequencies, the surface wave penetrates far into the ground and its wavenumber follows the Rayleigh wavenumber of the underlying stiff soil (which is slightly larger than the shear wavenumber). At around 20 Hz the wavenumber of the first surface wave increases sharply, tending at higher frequencies towards the Rayleigh wavenumber of the upper soft layer. This occurs when the surface wave becomes more localised near the ground surface as the wavelength reduces [1]. This frequency (20 Hz here) will be referred to in the remainder of the paper as the ‘cut-on’ frequency of the upper layer. For the deeper surface layer, not shown here, the same phenomena occur but at lower frequencies, with the first surface wave becoming localised in the upper layer from around 10 Hz [7].

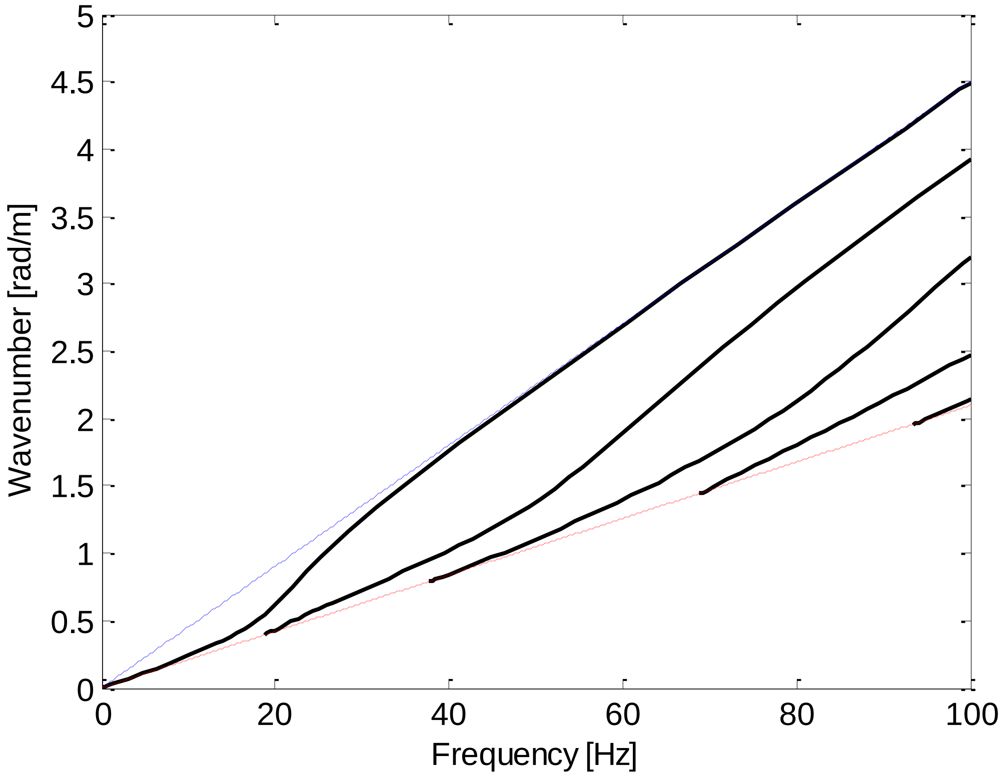

Fig. 3. Dispersion plot (—) of surface waves for soil with a 3 m upper layer and properties as given in Table 1. Blue dashed line, Rayleigh wave in a half-space of the upper soil material; red dash-dot line, shear wave in the lower soil material (reproduced from Soil Dynamics and Earthquake Engineering 79 (2015) D.J. Thompson et al., Mitigation of railway-induced vibration by using subgrade stiffening 89-103, with permission from Elsevier) [7].

The vertical displacement of the ground at a distance of 24 m predicted using the FE/BE model is shown in Fig. 4 for the four grounds. Results are shown for the case without the track (forces acting on the ground surface) as well as the case with the track that is considered throughout the rest of the paper. As in the insertion loss calculations in Sections 3-5, the excitation here consists of a ‘line load’ formed of 16 incoherent vertical point loads on each rail (or on the ground surface) corresponding to the locations of the axles of a four-car train (see Table 3 for the vehicle dimensions). This approximates more closely the excitation due to a train than a single point load would [7].

For the softer half-space, the response is 16 times higher than for the stiffer soil due to the difference in shear wave speed (factor of 4). However, if a layer of soft soil is introduced over the stiffer half-space, the response follows that of the stiff soil at low frequencies before rising sharply to a level similar to that of the soft half-space. For the 3 m soft layer it rises between 10 and 20 Hz, corresponding to the cut-on frequency identified in Fig. 3, while for the deeper soft layer it rises between 5 and 10 Hz.

The inclusion of the track has negligible effect at low frequencies but leads to a considerable reduction in the response at 80 Hz for the cases with a soft surface layer, including the soft half-space. This is caused by the width of the track. At this frequency the wavelength of the Rayleigh waves in this softer soil material is approximately 2 m. The two rails are 1.5 m apart and, although the sleeper is 2.5 m wide, due to its flexibility the equivalent width of the loading region is approximately 2 m. The track distributes the load such that the transmission through the soft upper layer becomes less important at this frequency than in the case of point loads directly acting at the soil’s surface. The wave transmission through the stiffer underlying half-space remains, which is less affected by the barrier.

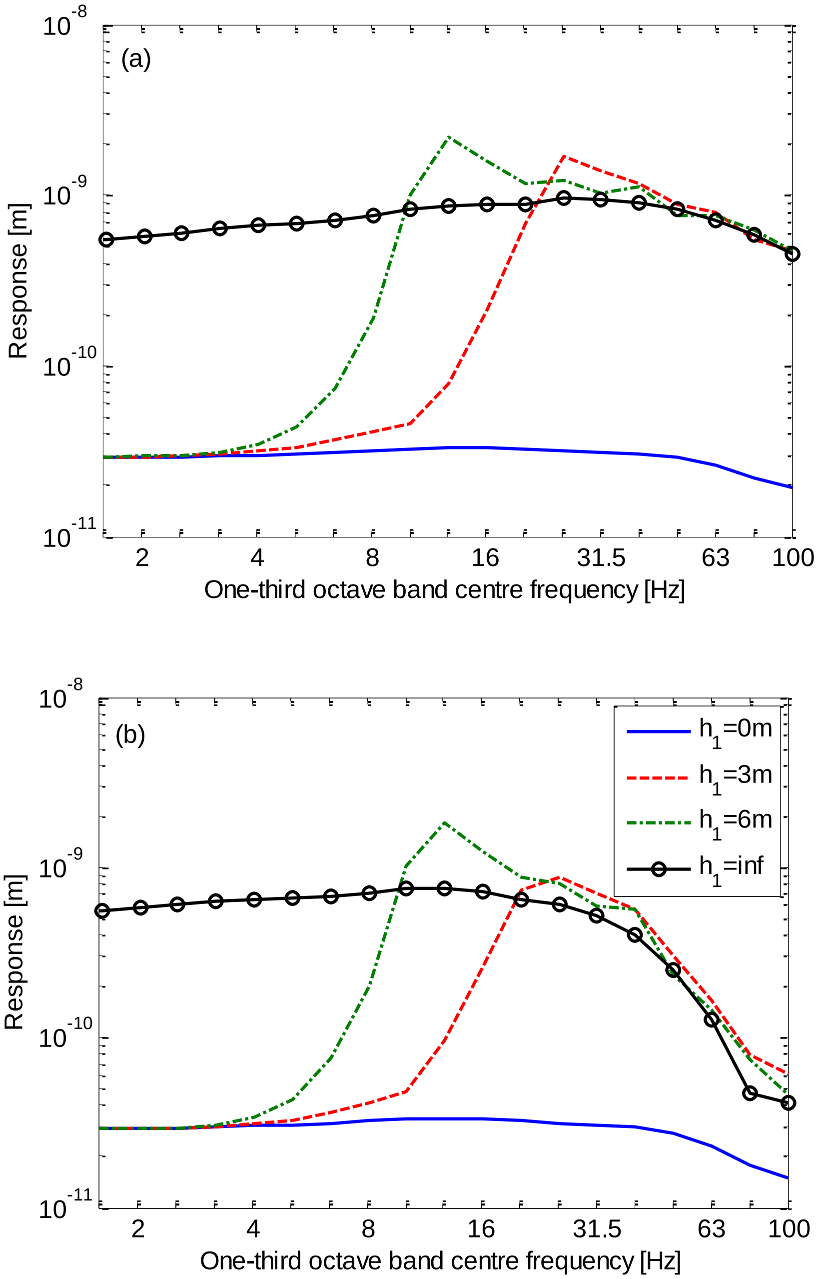

Fig. 4. Response at 24 m away from the ‘line’ source applied (a) on the ground surface; (b) on the track. The depth of the soft upper layer is h_1.

2.4 Vibration due to passing trains

Following the same procedure as in [7], to determine the effects of the trenches on the vibration observed in practice, examples of the vibration due to passing trains have been calculated using the analytical wavenumber domain model of Sheng et al [54]. The results are expressed as the average spectrum of vibration at certain distances from the track during the train pass-by. In the present work, as in [7], these will then be combined with the insertion loss spectra rather than including the moving train directly in the 2.5D FE/BE model.

The parameters assumed for the train are listed in Table 3; this is a typical four-car multiple unit passenger train. The unevenness spectrum is assumed to be the FRA class 3 roughness [55]. The unevenness is assumed to be identical (and in phase) on each rail; this is reasonable for wavelengths longer than around 1 m as the unevenness in this range corresponds to the track geometry. The track is assumed to consist of UIC60 rails, supported by rail pads on monoblock concrete sleepers and a ballast layer with a depth of 0.30 m. No sub-ballast, form layer or embankment is included. The parameters used for the track are listed in Table 2.

Table 3. Vehicle parameters used.

Car body mass [kg]	40,000
Car body pitching moment of inertia [kg.m^2]	2.0×10^6
Bogie mass [kg]	5000
Bogie pitching moment of inertia [kg.m^2]	6000
Unsprung wheelset mass [kg]	1800
Total axle load [N]	1.401×10^5
Bogie wheelbase [m]	2.7
Distance between bogie centres [m]	19.0
Overall vehicle length [m]	26.6
Number of vehicles	4
Primary suspension stiffness [N/m]	2.4×10^6
Primary suspension viscous damping [Ns/m]	3.0×10^4
Secondary suspension stiffness [N/m]	6.0×10^5
Secondary suspension viscous damping [Ns/m]	2.0×10^4
Contact stiffness (per wheel) [N/m]	1.46×10^9
Train speed [km/h]	250

The vibration results are shown in Fig. 5 in the form of one-third octave band velocity levels. This shows the vertical ground velocity at 24 m from the track centreline for each of the soil conditions listed in Table 1. It is expressed as the average vibration spectrum over the length of the train. From these results it can be seen that the vibration level increases significantly at the cut-on frequency associated with the upper layer of the layered grounds, i.e. approximately 10 Hz for the 6 m deep layer and 20 Hz for the 3 m layer. In both cases the level rises to become similar to that of the half-space of soft material. These vibration spectra will be used in the next sections together with insertion loss results obtained using the 2.5D FE/BE model to estimate the effect of trenches and soft barriers on train-induced vibration.

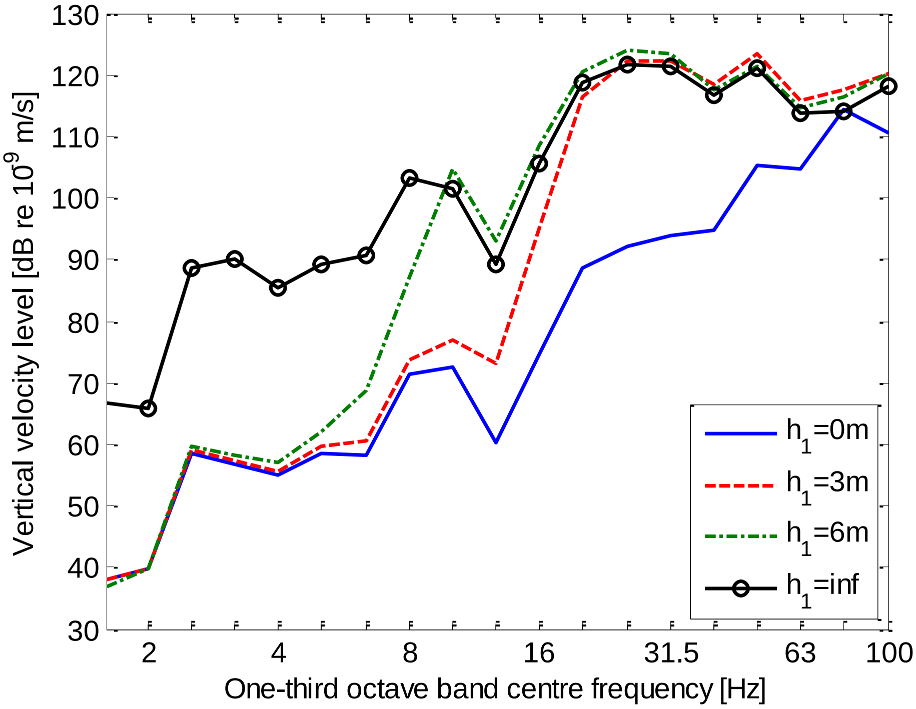

Fig. 5. Predicted vibration due to passing trains at 24 m from the track for a train speed of 250 km/h. h_1 denotes the depth of the upper soil layer.

3. OPEN TRENCH

3.1 Trench properties

In this section an idealised rectangular open trench is introduced at a distance of 8 m away from the track. Trenches with depths of 1.5, 3 and 6 m are considered; the width is set to 0.5 m unless otherwise stated. A typical FE/BE mesh for this case is shown in Fig. 6, in this case for a ground with a 3 m layer depth and a 3 m deep trench. As before, the ground surface is meshed with boundary elements having a node-to-node spacing of 0.25 m. To reduce corner effects, smaller elements (node-to-node distance of 0.025 m) are used to mesh the ground within 0.2 m on each side of the corners.

In each case, unless otherwise stated, the insertion loss (vibration level reduction at each frequency due to the introduction of the trench) is calculated at the ground surface at 24 m from the track centreline. The excitation consists of 16 incoherent point loads on each rail at the locations of the axles of the four-car train.

Fig. 6. Geometry of the 2.5D model for an open trench next to a railway track. Only part of the free surface and layer interface are shown.

3.2 Effect of soil layering and trench depth

Fig. 7(a,d) shows the insertion loss of an open trench in a homogeneous half-space. It is clear from these results that the insertion loss is negligible at low frequencies and rises above a certain frequency which depends on the depth of the trench and the soil stiffness. A deeper trench is thus able to attenuate Rayleigh waves with longer wavelengths (i.e. lower frequencies). This can be seen in the results in Fig. 7(a,d): by increasing the depth of the trench the effective frequency range extends to lower frequencies and the attenuation at higher frequencies is improved. The insertion loss is also larger for a softer ground, Fig. 7(d), than for stiff ground, Fig. 7(a). These results agree with the criterion [12] that an open trench is effective when the depth is at least 0.6 times the Rayleigh wavelength.

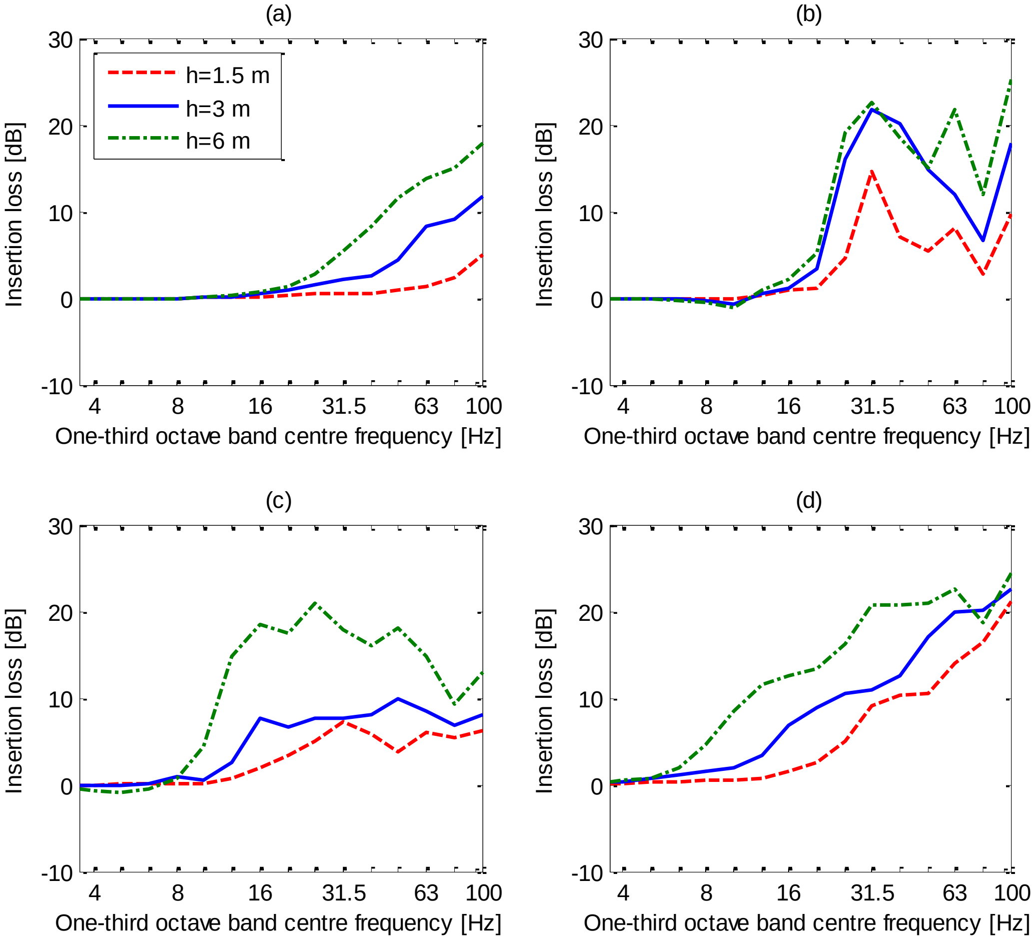

Fig. 7. Insertion loss at 24 m away from the excitation with 0.5 m wide open trench of various depths *h*. The depth of the soft upper layer is: (a) h_1 = 0 m, (b) h_1 = 3 m, (c) h_1 = 6 m and (d) h_1 = infinite. Response to a line source; calculated including the track.

Results for a layered ground are also shown in Fig. 7. As well as the trench depth, the performance is now also influenced by the depth of the soil layers. Above the cut-on frequency of the upper layer (20 Hz for the 3 m layer and 10 Hz for the 6 m layer), the waves are constrained within this layer and the trench provides attenuation. This attenuation is greatest when the depth of the trench is sufficient to penetrate fully the soft upper layer. As seen in Fig. 7(b), increasing the depth of the trench has little effect on the cut-on frequency of the insertion loss but the effectiveness of the trench improves. Once it penetrates through the upper layer (3 m deep), a further increase in the depth of the trench from 3 m to 6 m has little additional benefit on the performance. For the 6 m deep soil layer, however, Fig. 7(c) shows that a 6 m trench has a considerably better performance than a 3 m one.

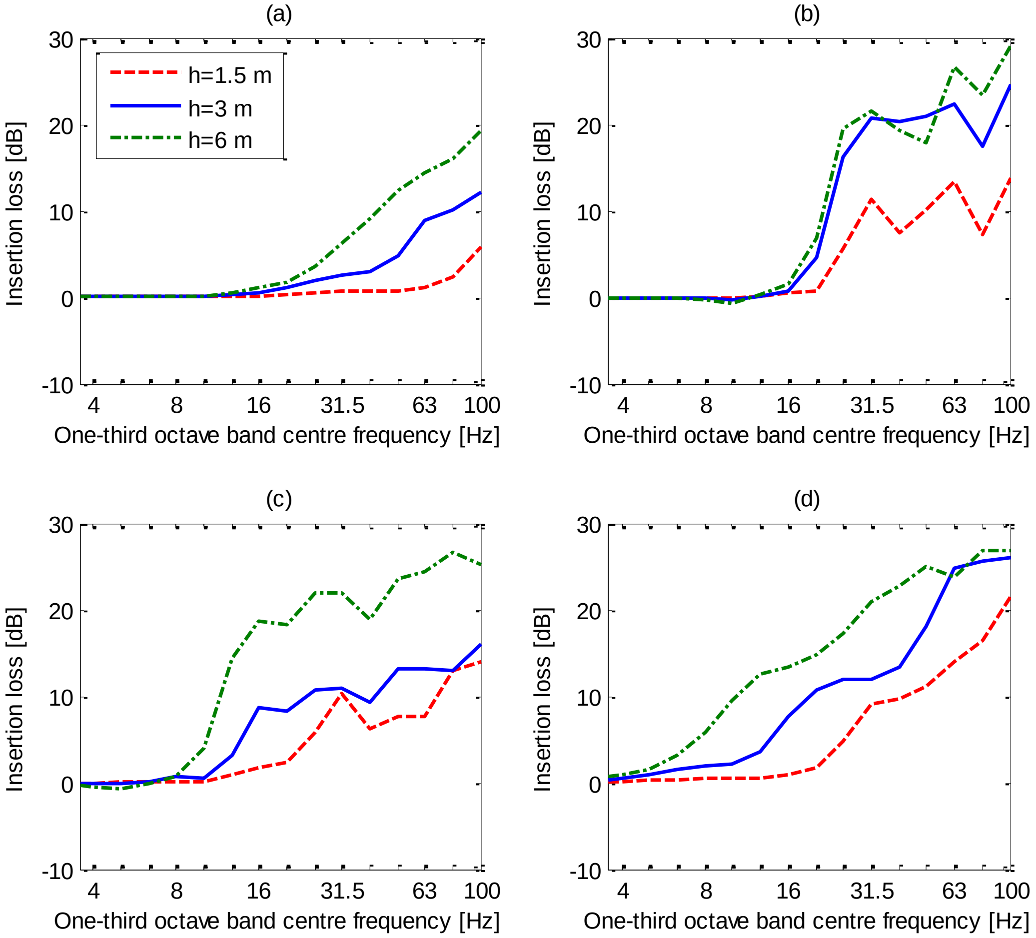

Fig. 8. Insertion loss at 24 m away from the excitation with 0.5 m wide open trench of various depths *h*. The depth of the soft upper layer is: (a) h_1 = 0 m, (b) h_1 = 3 m, (c) h_1 = 6 m and (d) h_1 = infinite. Response to a line source; no track included.

To illustrate the effect of the track, Fig. 8 shows results equivalent to Fig. 7 where the track has been omitted from the model and the forces have been applied directly to the ground surface. Comparing the results with those in Fig. 7 it can be seen that the insertion loss is affected by the presence of the track above about 30 Hz, particularly for the cases with a layered ground where the trench penetrates through the upper layer. At first sight this is surprising as the response at the track is unaffected by the trench. However, the results of Fig. 4 indicated that the presence of the track leads to a reduction in the response at 24 m at around 80 Hz due to the effect of the load distribution by the track. For the layered ground, the response at 80 Hz is therefore likely to be more affected by propagation through the underlying half-space. This component of the response will be unaffected by a 3 m deep trench, leading to a reduction in the insertion loss.

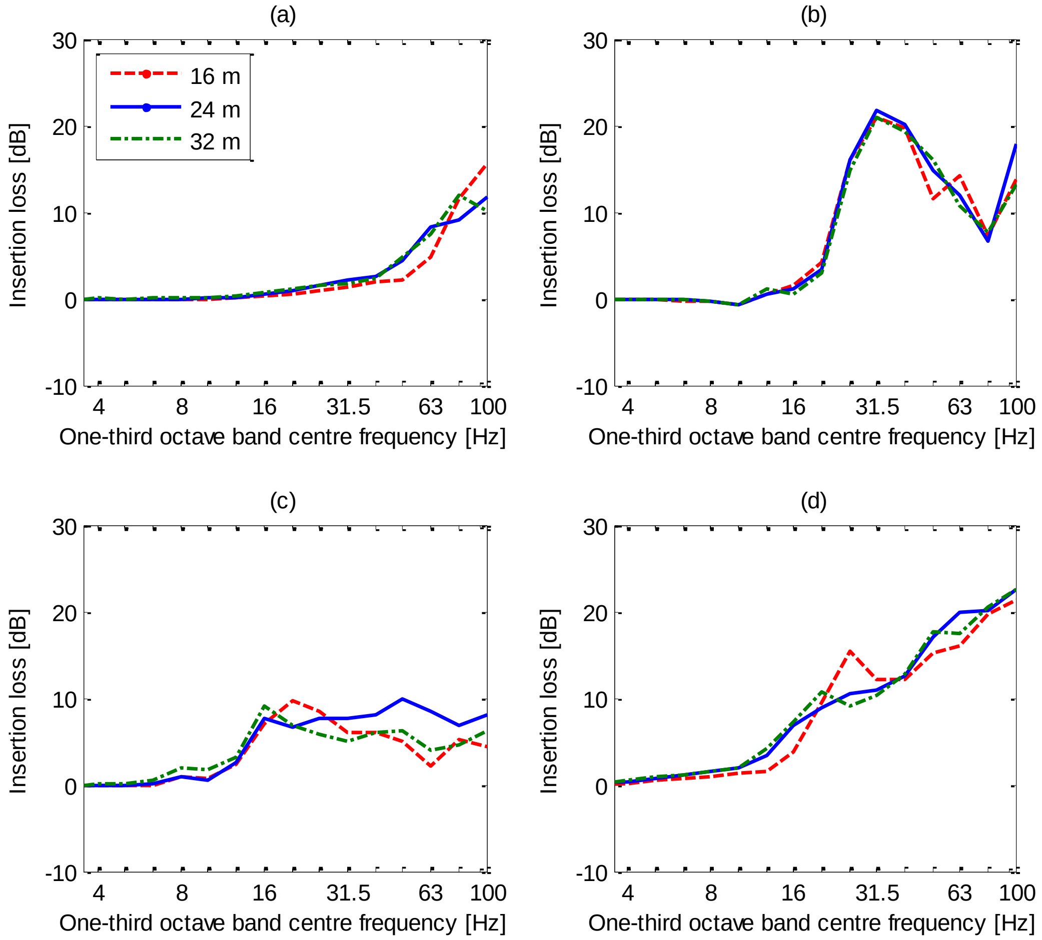

Fig. 9. Insertion loss at different distances away from the excitation with 0.5 m wide open trench of 3 m depth. The depth of the soft upper layer is: (a) h_1 = 0 m, (b) h_1 = 3 m, (c) h_1 = 6 m and (d) h_1 = infinite. Response to a line source; calculated including the track.

Fig. 9 shows results at different distances from the excitation. Although there are some small differences between the results, the same trends are seen at each distance away from the excitation.

3.3 Effect of trench width

The effect of the trench width is shown in Fig. 10, in this case only for the 3 m deep ground layer and 3 m deep trench. A wider trench does not affect the cut-on frequency of the insertion loss but can give a slightly improved insertion loss at higher frequencies. Increasing the width can thus be a way to increase further the insertion loss in the region where the trench is effective, at least for this case where the trench cuts through the upper layer. As before, the insertion loss drops at 80 Hz for the reasons described in the previous section.

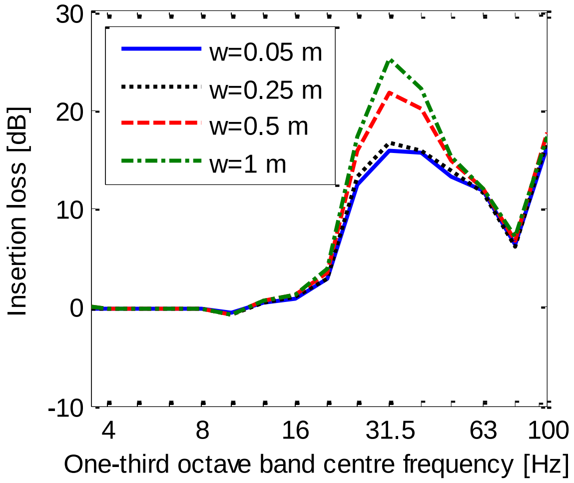

Fig. 10. Effect of varying the width *w* of a 3 m deep trench on the insertion loss at 24 m, for two-layer ground where the depth of soft upper layer is h_1 = 3 m. Response to a line source; calculated including the track.

3.4 More practical designs

As a deep rectangular trench is difficult to construct and maintain in practice, trenches with sloping side(s) have been proposed as an alternative [13]. The performance of four different geometries are compared here, as shown in Fig. 11. The first is a rectangular trench with both sides at 90° (vertical) as above. This is compared with a trench with both sides inclined at 45°; a trench with both sides inclined at 60° from the horizontal; and a trench with the side furthest from the track at 45° and the side nearest the track vertical. Their performance is compared in Fig. 12 for the soil with 3 m deep soft upper layer. The differences between the results for the different geometries can be seen to be fairly small. These results suggest that, for most situations, a trench with sloping sides can be used instead of rectangular trench to achieve insertion losses that are similar.

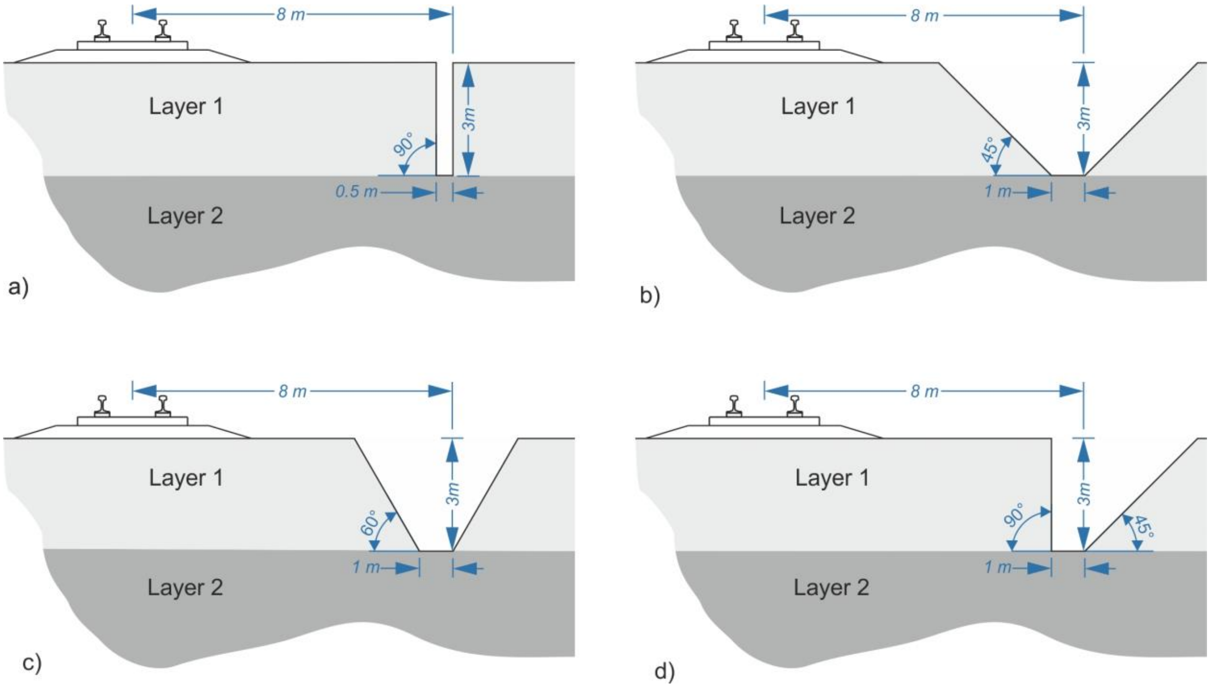

Fig. 11. Conditions considered for trenches with sloping sides.

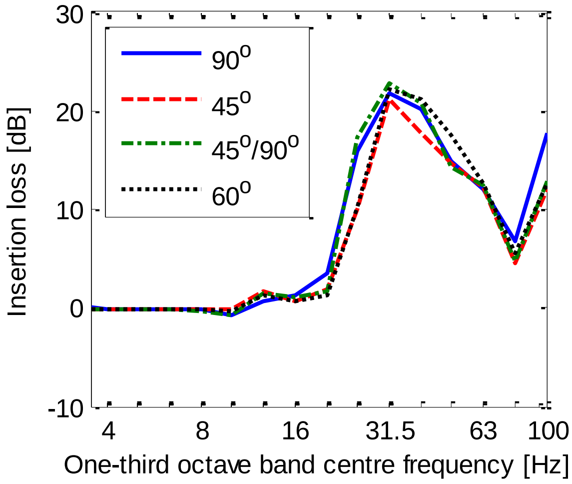

Fig. 12. Effect of varying angle of the sides of a 3m deep trench on the insertion loss at 24 m, for two-layer ground where the depth of soft upper layer is h_1 = 3 m. Response to a line source; calculated including the track.

3.5 Effect on train vibration

Combining the insertion losses obtained in this section with the train vibration spectra shown in Fig. 5 allows an estimate to be made of the effect of the trench on the train vibration.

Example results are shown in Fig. 13 for all four soil conditions. The overall velocity levels, given by the sum of the mean-square values in the frequency range 1.25 to 100 Hz, are listed in Table 4 along with the reductions relative to the case with no trench.

Compared with the results in Fig. 5, it can be seen that, for all cases except the 6 m deep surface layer, a 3 m deep trench gives considerable reductions in the levels in the frequency region where they were highest. As seen in Table 4, this corresponds to reductions of 7 to 11 dB in overall level. Increasing the trench to 6 m deep so that it penetrates the 6 m deep layer also gives good reductions for this case and increases the benefit for all soils to between 13 and 17 dB. Although the insertion loss was smallest for the stiff half-space ($h_1 = 0$ m), the initial vibration was lowest for this soil; therefore after application of the trench the vibration remains the lowest for this soil.

Table 4. Overall velocity levels at 24 m from track during passage of a train at 250 km/h in dB re 10^{-9} m/s. Δ is the level difference in dB relative to the case with no trench.

	$h_1 = 0$ m	$h_1 = 3$ m	$h_1 = 6$ m	$h_1 = \infty$
No trench	116.6	129.4	129.9	128.2
3 m trench	108.0	117.0	122.1	116.4
Δ	8.6	12.4	7.8	11.8
6 m trench	102.3	114.6	113.7	110.7
Δ	14.3	14.8	16.2	17.5

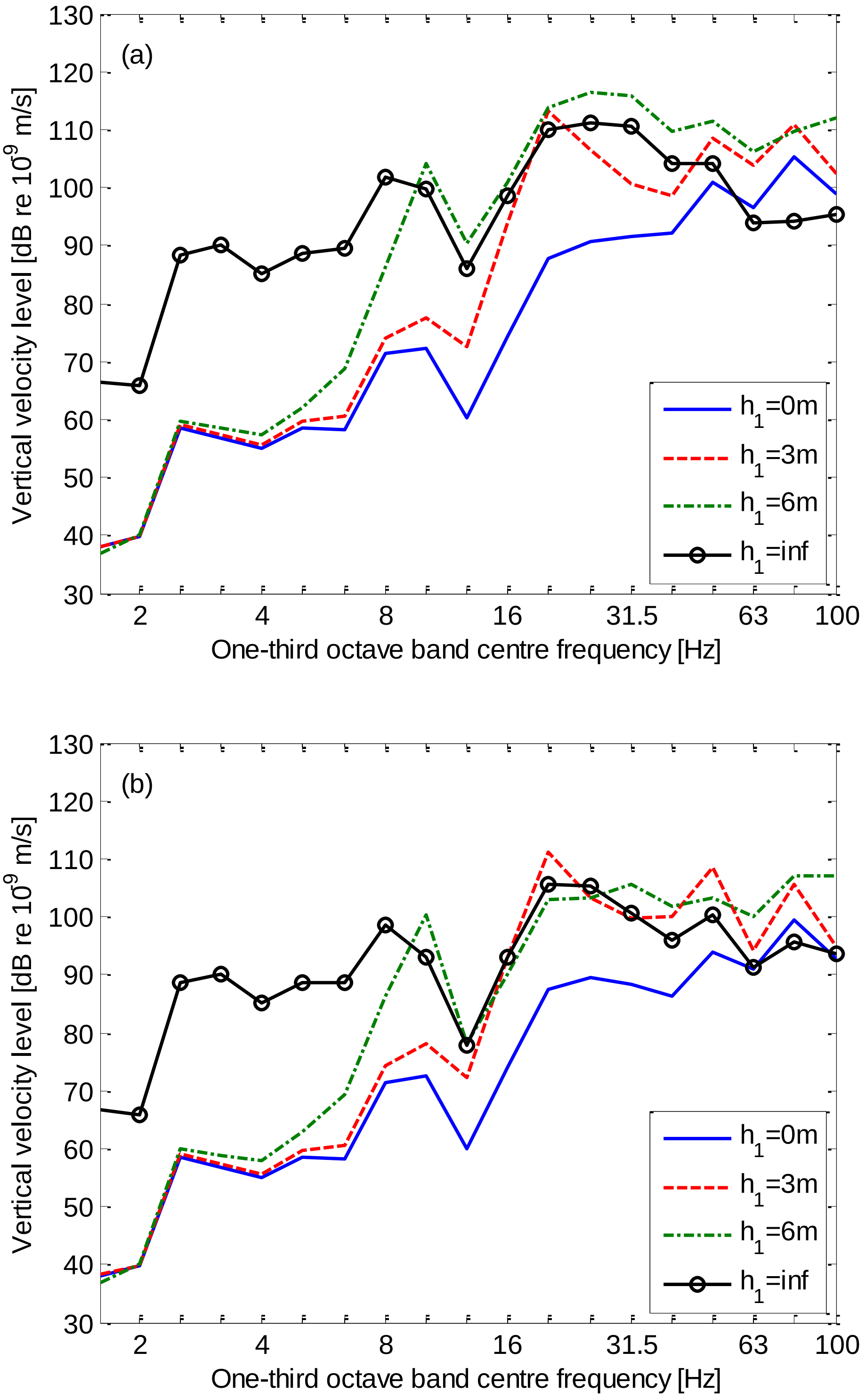

Fig. 13. Predicted vibration due to passing trains at 24 m from the track for a train speed of 250 km/h with a 0.5 m wide open trench: (a) 3 m deep; (b) 6 m deep. h_1 denotes the depth of the upper soil layer.

4. SOFT-FILLED BARRIER

4.1 Properties of fill material

Although an open trench has been shown to achieve insertion losses of 10-20 dB for a layered ground above the cut-on frequency of the upper layer, in practice an open trench of depth 6 m will not be stable. One method to ensure its stability is to fill the trench with a material that is relatively soft compared with the surrounding soil, yet is sufficiently stiff to sustain the confining pressure of the soil. In this section, therefore, consideration is given to a trench filled with a soft barrier material. Only the ground with the 3 m layer depth is considered in this section.

Table 5. Dynamic properties of fill materials.

	E [MPa]	ν	ρ [kg/m³]	c_s [m/s]	c_p [m/s]	ρc_p [kg/m²s]
A	1.0	0.4	700	22.6	55.4	38780
B	0.5	0.4	350	22.6	55.4	19390
C	0.25	0.4	175	22.6	55.4	9695
D	1.0	0.4	2800	11.3	27.7	77560
E	0.25	0.4	700	11.3	27.7	19390

Various notional but realistic materials are considered, the properties of which are listed in Table 5. Material A is based on a real material (normally used as an elastic layer in acoustic floating floors) that was considered for use in a field test within the RIVAS project [53]. Of the others, B and C are chosen to vary the Young's modulus without changing the shear wavespeed, while D is chosen to have the same Young's modulus as A and a reduced shear wavespeed. Finally material E has this same reduced wavespeed but with the same Young's modulus as material C; it also has the same values of impedance ρc_s (and ρc_p) as material B. Compared with the soil, both the Young's modulus and the impedances of all the materials are at least a factor of 100 smaller. Note that materials with a density less than 1000 kg/m³ may be impractical in saturated soils.

Fig. 14 shows a typical example of the structure considered in the study and the mesh used in the numerical model. A buried soft barrier is placed at 8 m from the track and penetrates through the first layer of soil. As in the previous sections, the ground is meshed using 3-node boundary elements with a maximum node-to-node distance of 0.25 m and smaller elements were used on each side of the corners. The barrier is modelled using 8-node rectangular finite elements. The node-to-node distance in the height direction of the fill material was set to be 0.05 m; a single element was used in the width direction. Unless otherwise stated the width is 0.05 m. Although this is very thin, a method for constructing such a thin soft barrier is described in [56]. As in Section 3, the insertion loss is calculated using 16 incoherent point loads on each rail, corresponding to the positions of the wheelsets of the train.

Fig. 14. Geometry of the 2.5D model for buried wall barrier next to a railway track. Only part of the free surface and layer interface are shown.

4.2 Effect of barrier depth

In Fig. 15(a) the insertion loss calculated for a 50 mm wide soft barrier using material A is compared with that for an open trench. Although the shape is similar, the attenuation between 31.5 and 80 Hz is limited to a maximum of around 6 dB as a result of transmission through the soft barrier material. Results for different depths are shown in Fig. 15(b). The same vertical scale is used here (and in subsequent graphs) for direct comparability with the results in the previous section. In the same way as for the open trench, a barrier that does not cut through the whole upper soil layer has a reduced effectiveness, whereas once the barrier penetrates through the whole of this layer there is little additional benefit in making it deeper.

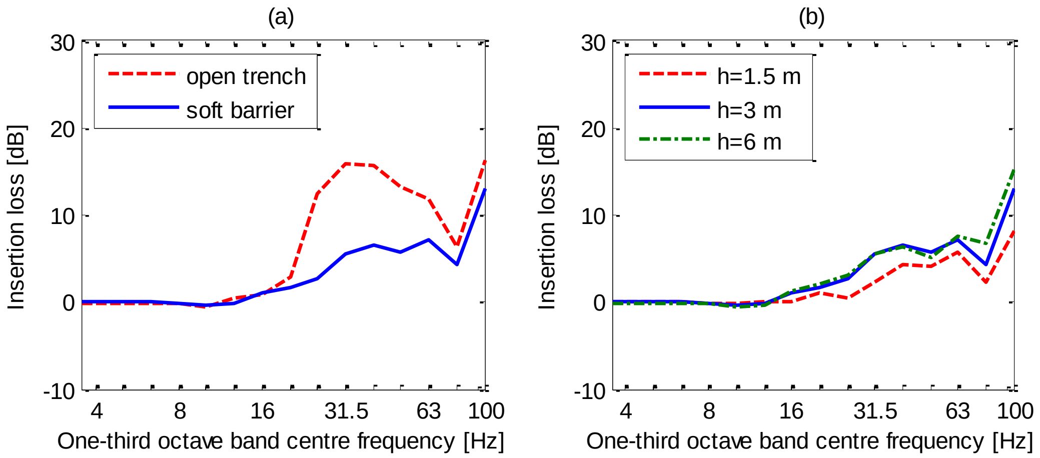

Fig. 15. Insertion loss at 24 m for 50 mm wide soft barrier using material A, for two-layer ground where the depth of soft upper layer is h_1 = 3 m. (a) 3 m deep soft barrier compared with open trench of the same width. (b) Soft barrier with depth h = 1.5, 3 and 6 m. Response to a line source; calculated including the track.

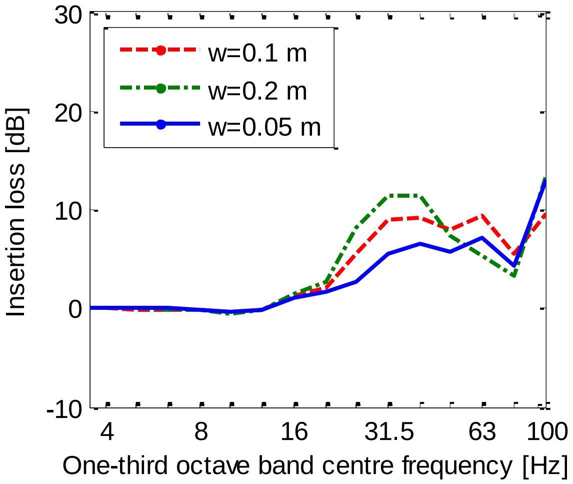

Fig. 16. Insertion loss at 24 m for 3 m deep soft barrier using material A with various widths w, for two-layer ground where the depth of soft upper layer is h_1 = 3 m. Response to a line source; calculated including the track.

4.3 Effect of barrier width

Fig. 16 shows results for 3 m deep barriers of different widths using material A. Whereas the width had only a small effect for an open trench, see Fig. 10, clearly increasing the width gives additional benefit as the barrier stiffness is reduced. The 200 mm wide barrier has a slight reduction in performance around 60 Hz where the width is equal to half the shear wavelength of the fill material.

4.4 Effect of ground material properties

Changing the shear wave speed of the top layer of soil from 150 m/s to 100 m/s or 200 m/s gives results as shown in Fig. 17. From the results, only small differences are seen. The main differences are that the cut-on frequency of the barrier effect occurs at a lower frequency for a lower wavespeed, whereas the maximum insertion loss is smaller for softer soil. This occurs despite the fact that the elastic moduli of the barrier are a factor of 100 smaller than those of the soil.

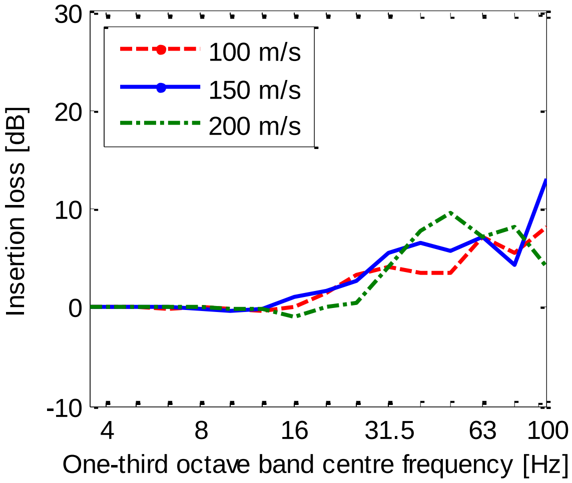

Fig. 17. Effect of the shear wave speed of the upper soil layer with depth h_1 = 3 m on the insertion loss at 24 m of 3 m deep, 50 mm wide soft barrier using material A. Response to a line source; calculated including the track.

4.5 Effect of barrier material properties

The result of changing the Young’s modulus of the fill material is shown in Fig. 18(a). Reducing the Young’s modulus (changing from material A to C) gives an improvement in the insertion loss, similar to increasing the width of the soft barrier as shown in Fig. 16. However, changing the shear wavespeed of the fill material while keeping the Young’s modulus fixed has a negligible effect on the insertion loss: compare materials D and E in Fig. 18(b) with A and C in Fig. 18(a). From these results, and the effect of changing the width in Fig. 16, it is clear that it is the stiffness of the layer that is the dominant parameter, not its impedance as suggested by Massarsch [14] and used by other authors, e.g. [16, 40]. In particular, materials B and E (see Table 5) have the same impedance but different insertion loss. The relevance of the stiffness will be demonstrated analytically in the next section.

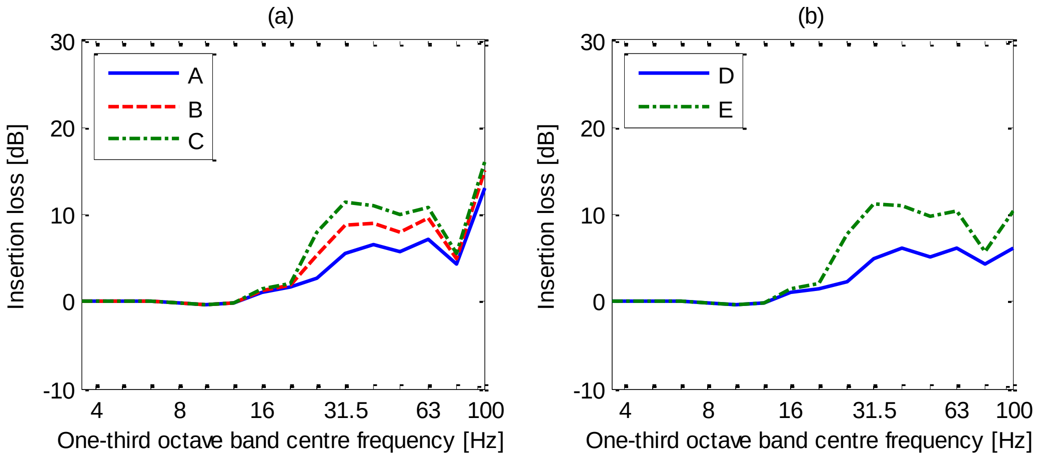

Fig. 18. Insertion loss at 24 m for 3 m deep, 50 mm wide soft barrier, for two-layer ground where the depth of soft upper layer is h_1 = 3 m. Material properties as in Table 5. (a) c_s = 22 m/s, E = 1, 0.5 and 0.25 MPa, (b) c_s = 11 m/s, E = 1 and 0.25 MPa. Response to a line source; calculated including the track.

4.6 Transmission through infinite soft barrier

To approximate the transmission through the barrier, consider an infinite soft-filled barrier in a homogeneous soil subject to an incident plane wave at normal incidence. Transmission will occur, in practice, due to both compressional waves and shear waves but for simplicity only compressional waves are considered here. The soil can be characterised by its P-wave elastic modulus $K_s = \lambda + 2\mu$, and compressional wave speed c_p. The transmission problem is similar to a one-dimensional waveguide with a soft insert. Write A_1 as the complex amplitude of the incident wave, A_2 for the reflected wave and A_3 for the transmitted wave. The soft barrier has thickness d and P-wave elastic modulus K_b; for a Poisson's ratio of 0.4 the modulus K_b is $1.43E$. The barrier applies a compressional stress to the soil on either side of

$$\sigma = \frac{K_b}{d}(A_1 + A_2 - A_3) \tag{1}$$

This can be equated to the internal stress in the soil on either side of the barrier

$$\sigma = K_s(-ikA_1 + ikA_2) = -K_s ikA_3 \tag{2}$$

where $k = \omega/c_p$ is the wavenumber of compressional waves in the soil. Solving these equations yields a power transmission coefficient

$$\tau = \frac{1}{1+\left(\frac{kK_s d}{K_b}\right)^2} \tag{3}$$

Fig. 19 shows the transmission loss, $10.\log_{10}(1/\tau)$, for the parameters considered here for a soft layer of width 0.05 m and soil with compressional wavespeed 298 m/s as in Table 1. The presence in equation (3) of the factor K_b/d shows the influence of the stiffness per unit area of the barrier on the transmission loss. The results for shear waves (not shown) are similar although 3 dB higher at high frequencies due to the lower wavespeed and hence larger wavenumbers.

In the case of the finite soft-filled barrier, vibration will be transmitted past the barrier due to a combination of diffraction under the barrier and transmission through the barrier itself. The results in Fig. 19 approximate the latter effect whereas the diffracted field corresponds to the result for the open trench, see Fig. 10 for the result for a trench of width 0.05 m. It can be seen that the results of Fig. 18 follow the trends of the minimum of these two results.

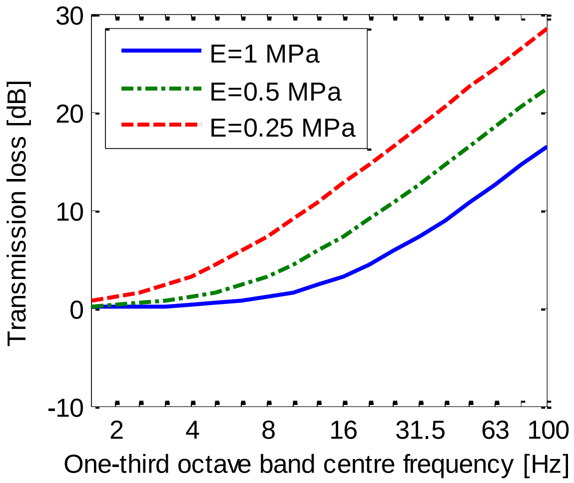

Fig. 19. Transmission loss of an infinite soft-filled barrier in a soil due to an incident compressional wave with velocity 298 m/s.

4.7 Effect on train vibration

Applying the insertion losses from Fig. 18(a) to the train vibration spectrum of Fig. 5, the resulting train vibration in the presence of a soft-filled barrier can be estimated. The results are shown in Fig. 20. Although the soft-filled barrier only has an effect in a limited part of the frequency range, this corresponds to the region in which the vibration is highest, due to the ground layering. The overall vibration velocity level (the sum over the range 1.25 to 100 Hz) is thereby reduced by 4.9 dB for a Young's modulus of 1 MPa, 7.0 dB for 0.5 MPa and 8.7 dB for 0.25 MPa. These may be compared with a reduction of 11.0 dB that would be achieved by an open trench of the same width (0.05 m), which is included as a hypothetical case for the sake of comparison.

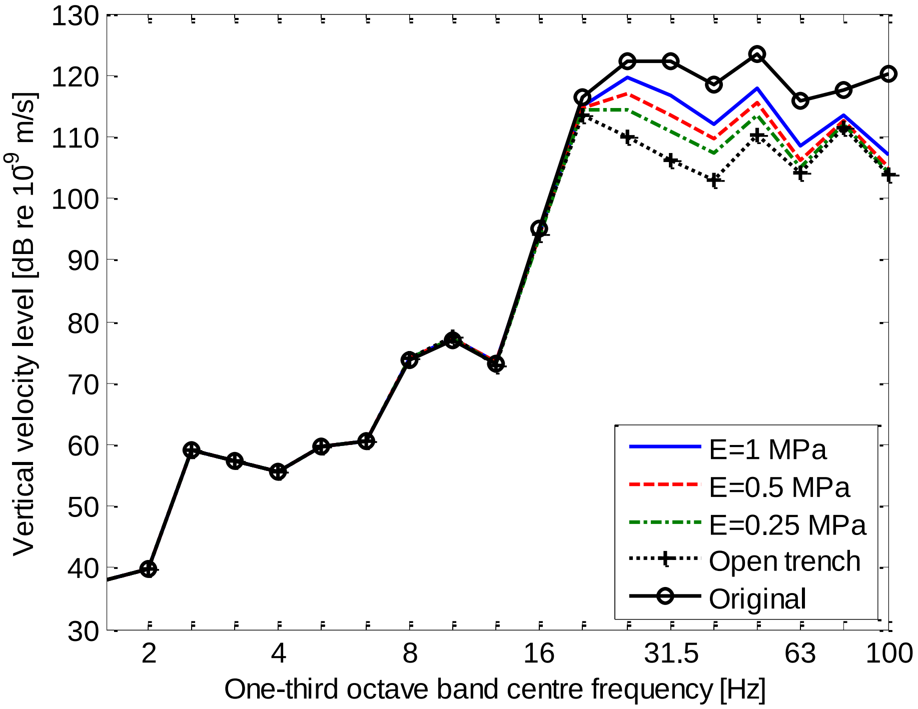

Fig. 20. Predicted vibration due to passing trains at 24 m from the track for a train speed of 250 km/h with a 3 m deep soft-filled barrier, width 0.05 m, various values of Young's modulus *E*; two-layer ground where the depth of soft upper layer is $h_1 = 3$ m.

5. CONSIDERATION OF OTHER SITE CONDITIONS

5.1 Soil parameters

In this section, the effects of the measures considered in this paper are determined for other soil conditions. The parameters from three sites are used at which field measurements were carried out during the RIVAS project [53]. The same sites have also been studied in [7, 11] which allows comparison of the effectiveness of different mitigation measures. The dynamic soil characteristics (layer thickness h, shear wave velocity c_s, compressional wave velocity c_p, density ρ, and material damping loss factors η) are shown in Table 6. The soil at the site at Horstwalde, Germany is a sandy soil to a depth of at least 10 m and is represented as a homogeneous half-space. The site at Lincent, Belgium is situated next to the high speed line between Brussels and Liège. At this site a shallow top layer is followed by a layer of fine sand (layer 2). Below this a sequence of hard arenite layers embedded in clay is followed by a sequence of fine sand and clay layers. In the model this is represented as two softer top layers with a total thickness of 4.1 m above a stiffer half-space. The high value for the compressional wave velocity of the underlying half-space is caused by saturation of the soil. This has similar characteristics to the layered soils considered in the earlier sections apart from the separation of the upper layer into two. The site of Furet, Sweden is located adjacent to the West Coast line between Gothenburg and Lund. This is a particularly soft soil site at which an inverse layering is present. The upper layer is relatively firm sand of depth 2 m below which is a 10 m deep layer of clayey silt followed by silty clay.

Table 6. Soil properties for the reference sites.

	Layer	h [m]	c_s [m/s]	c_p [m/s]	η [-]	ρ [kg/m³]
Horstwalde	1	∞	250	1470	0.05	1945
Lincent	1	1.4	128	286	0.088	1800
	2	2.7	176	286	0.076	1800
	3	∞	355	1667	0.074	1800
Furet	1	2	154	375	0.05	1800
	2	10	119	290	0.05	1850
	3	∞	200	490	0.05	1710

5.2 Open trench

Simulations using the same open trench (0.5 m wide, 3 m deep) as used in Section 3 were carried out at the three field sites. Insertion losses at three distances are shown in Fig. 21 for all three sites (based on a line source). For the homogeneous half-space Horstwalde, the effectiveness of the trench is mostly related to the Rayleigh wavelength as discussed in Section 3. In this case the criterion that the depth is at least 0.6 times the Rayleigh wavelength is met at about 50 Hz. At Lincent, with a soft soil layer over a stiffer sub layer, above a certain frequency surface waves are localised in a soft upper soil layer. In these conditions a trench is most effective if it can fully penetrate the upper soil layer. At Lincent the soft layer is effectively 4.1 m deep and therefore the 3 m trench does not fully penetrate. Nevertheless, the trench provides effective attenuation above around 16 Hz with a sharp peak at 25 Hz. Although the frequency range of effectiveness varies, the open trench can achieve a maximum insertion loss of between 10 and 20 dB at these two sites. At Furet, however, where the soil is soft down to a depth of 12 m, the 3 m trench only provides limited benefit, which reduces further as the distance to the receiver increases.

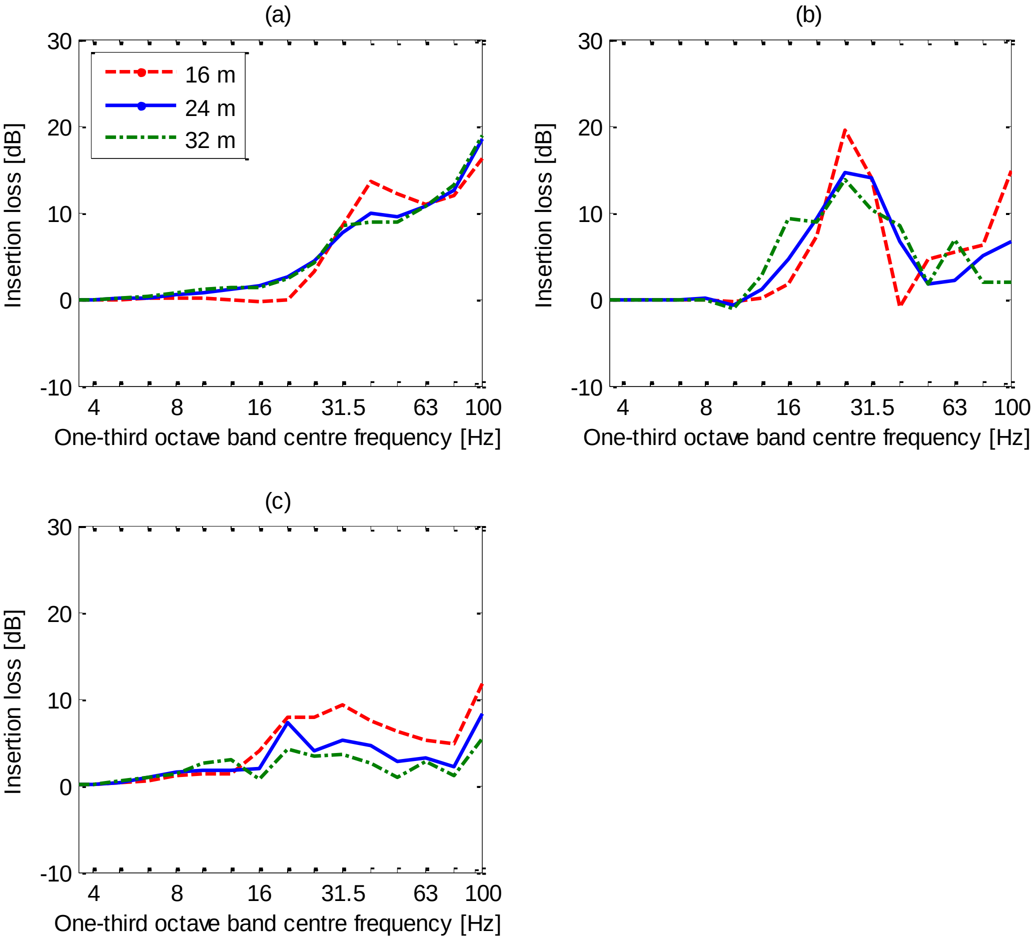

Fig. 21. Insertion loss of a 3 m deep, 0.5 m wide open trench for different sites. (a) Horstwalde, (b) Lincent, (c) Furet. Receiver is at 16 m (solid line), 24 m (dashed line) and 32 m (dash-dot line) from the track. Response to a line source; calculated including the track.

5.3 Soft-filled barrier

In Fig. 22 results are shown for these three sites for a 50 mm thick soft-filled barrier using material A of Table 5. At Horstwalde there is still significant reduction at frequencies above 40 Hz, although the benefit is less than for the open trench. At Lincent, however, the benefit of the open trench is almost completely lost, while at Furet the insertion loss remains small.

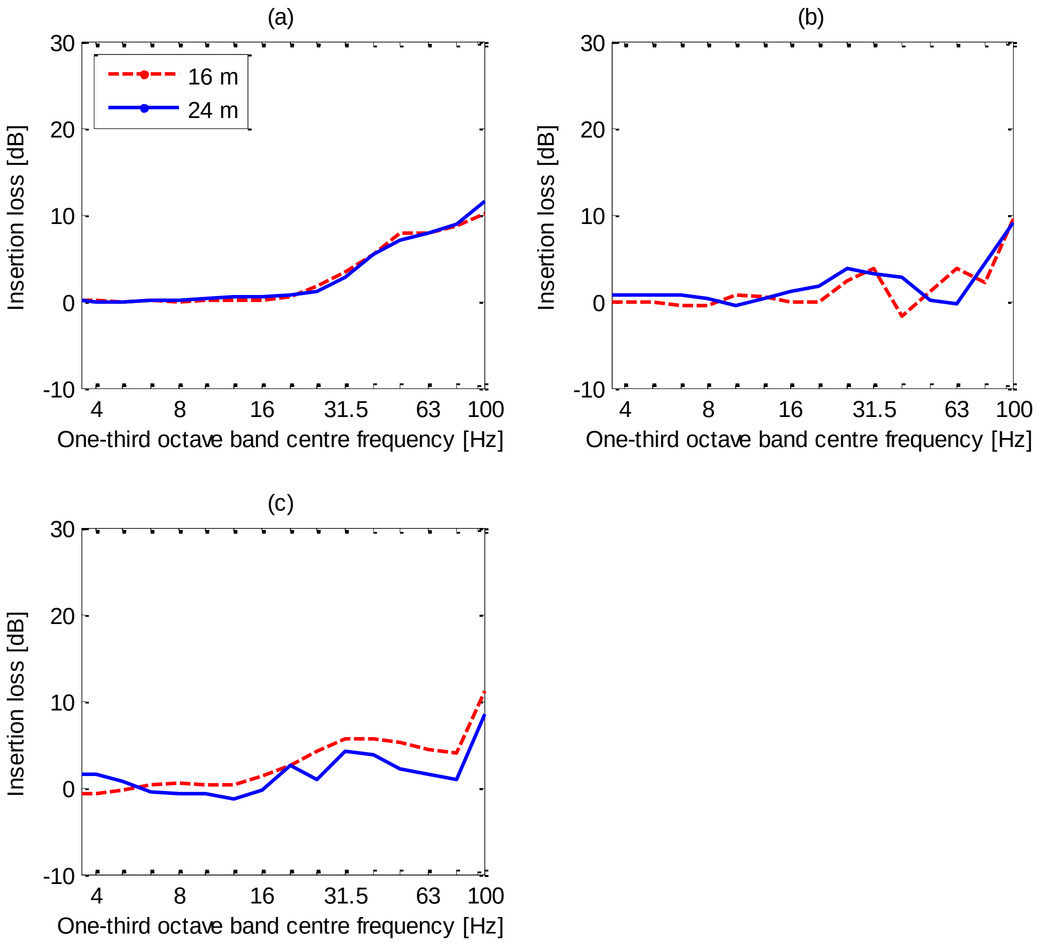

Fig. 22. Insertion loss of a 3 m deep, 50 mm wide soft-filled barrier for different sites. (a) Horstwalde, (b) Lincent, (c) Furet. Receiver is at 16 m (solid line) and 24 m (dashed line) from the track. Response to a line source; calculated including the track.

6. CONCLUSIONS

From this study using the 2.5D FE/BE method it has been shown that the idealised open trench is a very effective mitigation measure to reduce the ground-borne vibration. For a homogeneous ground the results are consistent with the established rule of thumb that, to be effective, the depth should be at least 0.6 times the Rayleigh wavelength. However for a layered ground, the performance is also influenced by the depth and stiffness of the soil layers. For a layered ground with a soft weathered layer above a stiffer sub-stratum, significant reductions can be achieved if the trench cuts through the upper layer, whereas increasing the depth further has negligible benefit. The frequency region in which the benefit is achieved corresponds to the region in which railway-induced vibration is maximum due to the effect of the ground layering. In such cases, an overall reduction of railway vibration of 10 dB was achieved, for example, by a 3 m deep trench in a soil with a 3 m deep surface layer.

Increasing the width of the open trench has a relatively small effect on the benefit. Of more practical interest are trenches with sloping sides or retaining structures. Calculations show that the performance of a trench with one or more sides sloping (at 45° or 60° to the vertical) is very similar to that of a vertical-sided trench.

It is found that it is important to include the track in the model when determining the insertion loss of the trenches. This is due to the fact that there is much less transmission through the upper surface layer when the effective width of the track equals the wavelength of shear waves; the residual transmission is mostly passing through the lower sub-stratum which is then not attenuated by the trench.

Filling the trench with a soft barrier material leads to significant reduction in the performance as vibration is transmitted through the barrier material as well as being diffracted beneath it. It has been shown that the important parameter is the stiffness per unit area of the barrier material, not its impedance as in the case where transmission at the interface between two semi-infinite media is considered. Despite the reduction in performance, a 3 m deep, 0.05 m wide barrier filled with a material of Young’s modulus 1 MPa gives a reduction of over 4 dB in the overall train-induced vibration for the example case considered. This can be improved by reducing the stiffness of the material or increasing its width or depth.

Finally, by considering several other sites with different soil conditions it is shown that the performance of both the open and soft-filled trench varies considerably between locations. It is therefore not possible to generalise the results from any study, numerical or field-based, but rather vibration mitigation is site-specific. This shows the importance of carefully designing mitigation measures taking account of the soil properties at the site in question.

ACKNOWLEDGEMENTS

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