Free Search in Multidimensional Space III

Kalin Penev(✉)

School of Media, Art and Technology, Southampton Solent University,
East Park Terrace, Southampton SO14 0YN, UK
Kalin.Penev@solent.ac.uk

Abstract. Various scientific and technological fields, such as design, engineering, physics, chemistry, economics, business, and finance often face multidimensional optimisation problems. Although substantial research efforts have been directed in this area, key questions are still waiting for answers, such as: What limits computer aided design systems on optimisation tasks with high variables number? How to improve capabilities of modern search methods applied to multidimensional problems? What are software and hardware constraints? Approaching multidimensional optimisation problems raises in addition new research questions, which cannot be seen or identified on low dimensional tasks, such as: What time is required to resolve multidimensional task with acceptable level of precision? How dimensionality reflects on the search space complexity? How to establish search process orientation, within multidimensional space? How task specific landscapes embarrass orientation? This article presents an investigation on 300 dimensional heterogeneous real-value numerical tests. The study aims to evaluate relation between tasks' dimensions' number and required for achieving acceptable solution with non-zero probability number of objective function evaluations. Experimental results are presented, analysed and compared to other publications.

Keywords: Free search · 300 dimensional optimisation

1 Introduction

Various scientific and technological fields, such as design, engineering, physics, chemistry, economics, business, and finance often face multidimensional optimisation problems [2]. Multidimensional optimisation problems, however, require sufficient computational resources. In the same time natural life suggests that capability to cope by finite and limited resources with infinite and unlimited environment and problems can be considered as an advanced characteristic of living systems. This article attempts to explore model of similar behaviour. It presents an investigation on 300 dimensional scalable heterogeneous real-value numerical tests optimisation. Due to a specific performance identified in earlier publications [7], optimisation method explored in this study is Free Search (FS) [6] only.

I. Lirkov et al. (Eds.): LSSC 2015, LNCS 9374, pp. 251–257, 2015.
DOI: 10.1007/978-3-319-26520-9_27

The aim of this investigation is to find answers of the questions: How to improve capabilities of modern search methods applied to multidimensional problems? What are software and hardware constraints? The study aims also to investigate specific for multidimensional optimisation research questions such as: What time is required to resolve multidimensional task with acceptable level of precision? How dimensionality reflects on the search space complexity? How to establish search process orientation, within multidimensional space? How task specific landscapes embarrass orientation?

For this purpose five scalable numerical tests are used - Ackley [1], Griewank [4], Michalewicz [5], Rosenbrock [9] and Step [3] test functions.

2 Test Problems

Criteria for tests selection are: - must be scalable for multidimensional format; - must be with heterogeneous landscape. Chosen numerical test are scalable and form different search spaces. All tests are transformed for maximisation.

2.1 Ackley Test

This test [1], know from the literature is widely used for search methods evaluation.

$$f(x) = a \exp\left[-b\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2\right)\right]^{1/2} + \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i)\right) - a - \exp(1) \quad (1)$$

where $a = 20$, $b = 0.2$, $c = 2\pi$. The maximum is $f_{max} = 0$, for $x_i = 0$, $i = 1, \ldots, n$. The search space borders are defined by $x_i \in (-32, 32)$.

2.2 Griewank Test

The test [4], is given by the following analytical expression:

$$f(x) = -\left(1 + \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right)\right) \quad (2)$$

where $x_i \in [-600.0, 600.0]$. The maximum is $f_{max} = 0$, for $x_i = 0$, $i = 1, \ldots, n$.

2.3 Michalewicz Test

Michalewicz test function is described [5] as global optimisation problem. Optimal value is dependent on dimensions number.

$$f(x) = \sum_{i=1}^{n}\sin(x_i)\left(\sin\left(\frac{ix_i^2}{\pi}\right)\right)^{2m} \quad (3)$$

where search space is defined as $x_i \in [0, \pi]$, $i = 1, ..., n$. For 300 dimensions maximum is unknown.

2.4 Rosenbrock Test

This function landscape is smooth flat hill with one optimal solution [9]. The test function is:

$$f(x) = -\sum_{i=1}^{n-1} \left[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right] \tag{4}$$

where $x_i \in [-2, 2]$, $i = 1, \ldots, n$. It has one maximum $f_{max} = 0$, for $x_i = 1$, $i = 1, \ldots, n$.

2.5 Step Test

Step test [3] introduces plateaus to the topology. The search process cannot rely on local correlation. Maximal are all locations, which belong to the plateau $x_i \in [2.0, 2.5)$. The maximum is dependent on dimensions number. The test function is:

$$f(x_i) = \sum_{i=1}^{n} \lfloor x_i \rfloor \tag{5}$$

where $x_i \in [-2.5, 2.5]$. For $n = 300$ maximum is $f_{max} = 600$, for $x_i \in [2.0, 2.5)$, $i = 1, \ldots, n$.

3 Optimization Method

Due to the abilities to produce acceptable results within feasible period of time identified in earlier publications [7,8], optimisation method selected for this study is Free Search (FS) [6] only.

3.1 Free Search

Free Search is adaptive heuristic method [6–8] for real coded optimisation. This section refines the description of some of its essential properties, published earlier. Optimisation process is organised in individual explorations within individuals' neighbour space [6]. In the beginning algorithm has no knowledge about the search space. First exploration is initial trial, which generates knowledge stored in a form of qualitative indicators related with evaluated locations. These indicators facilitate further explorations. Individuals develop sense to the indicators' quality. This sense is an original peculiarity of Free Search, which has no analogue in other methods. Individuals use their sense to locations quality for orientation within the search space.

Although individuals' sensibilities are highly uncertain a review of idealised general states of distribution such as uniform, enhanced and reduced sensibility related with locations' qualities can clarify the search process self-regulation. On initial stage locations quality and sensibility are uniformly distributed among low, medium and high levels. Individuals with low level of sensibility can select for start position any marked location. The individuals with high sensibility can select for start position marked locations with high quality and will ignore locations with low quality.

When marked locations quality highly differs and stochastically generated sensibility produces accidentally high values only, then the individuals will search around the area of the highest quality solutions. Such situations appear naturally. In this manner process converges to high quality locations. External addition of a constant or a variable to the sensibility could lead to an enforced enhancement of the sensibility. In this case all the individuals with enhanced sensibility will select and can differentiate more precisely locations with high quality and will ignore these with low quality. This could accelerate convergence to areas with high quality locations.

Other situation which naturally appears is when marked locations qualities are very similar and randomly generated sensibility is low. In this case individuals can select low quality marked locations with high probability, which indirectly will decrease the probability for selection of high quality marked locations. Subtraction of a constant or a variable from individuals' sensibility could make an enforced sensibility reduction. Individuals with reduced sensibility can select to explore around locations marked with low quality. As far as locations quality is independent on their position within the search space, similar quality locations could be remotely distributed. This facilitates divergence across the entire search space. Sensibility varies across all the individuals and during the optimisation process.

One of the objectives of this study is to evaluate how this manner of orientation performs for multidimensional space. For presented experiments Free Search operates with 10 individuals and explorations are 5 steps, for all experiments. The sense is random in the highest 10 % of the sensibility, and the neighbouring space varies from 0.5 to 1.5 with step 0.1 [6].

4 Experimental Methodology

Experimental Methodology aims to identify method's performance and level of precision for 300 dimensional tests limited to 3.10^8 objective functions evaluations. Each test function is evaluated in one series of 320 experiments, with start from random initial locations different for each experiment. Start locations are defined as:

$$x_{i0} = X_{min} + random_i(X_{max} - X_{min}) \tag{6}$$

where X_{max} and X_{min} are search space borders and $random_i(X_{max} - X_{min})$ generates random value between X_{max} and X_{min}, $i = 1, \ldots, n$. All variables are 300 dimensional vectors.

Rosenbrock test only is evaluated additionally one more series of 320 experiments limited to 3.10^9 objective functions evaluations.

5 Experimental Results

Achieved results are analysed for maximal and mean values, standard deviation and number of results with precision 0.01 from the maximal value.

On Tables 1, 2, and 3 FE denotes function evaluations number. Time periods in Table 3 are measured on processor Intel i7 3960x at 4.6 GHz and memory G. Skill Trident X at 1866 MHz, motherboard ASUS Rampage VI and solid state disk - SanDisk Extreme SSD SATA III.

6 Discussion

Analysis of experimental results suggests that Ackley, Michalewicz and Step tests can be resolved with 100 % probability with precision 0.001 for 3.10^8 function evaluations. Griewank test can be resolved with 82.81 % probability with precision 0.001 for 3.10^8 function evaluations.

Rosenbrock test cannot be resolved with acceptable level of precision for 3.10^8 function evaluations. Rosenbrock test is evaluated additional for 3.10^9 function evaluations. Rosenbrock test can be resolved with 76.56 % probability with precision 0.001 for 3.10^9 function evaluations, however the period of search is longer.

Comparison of the periods of search for these 300 dimensional tests and 200 dimensional tests publishes earlier [8] suggest that time increases higher than

Table 1. Experimental results from 320 experiments

Test	FE	Maximal results	Mean results	Standard deviation
Ackley	3.10^8	-0.000329198000	-0.000688728	0.000216832
Griewank	3.10^8	-0.000000215366	-0.004886839	0.008172175
Michalewicz	3.10^8	299.603000000000	299.595365600	0.003252990
Rosenbrock	3.10^8	-0.001858470000	-112.786252900	72.692261520
Rosenbrock	3.10^9	-0.000030781900	-0.090739686	1.472556809
Step	3.10^8	600	600	0

Table 2. Number and percentage of the results with precision above 0.01

Test	FE	Successful results	Successful results %
Ackley $\geq$ -0.00	3.10^8	320	100.00 %
Griewank $\geq$ -0.00	3.10^8	265	82.81 %
Michalewicz $\geq$ 299.59	3.10^8	320	100.00 %
Rosenbrock $\geq$ -0.00	3.10^8	4	0.39 %
Rosenbrock $\geq$ -0.00	3.10^9	245	76.56 %
Step = 600	3.10^8	320	100.00 %

Table 3. Periods of time for 3.10^8 objective functions evaluations

Test	FE	Time
Ackley	3.10^8	31 min
Griewank	3.10^8	48 min
Michalewicz	3.10^8	185 min
Rosenbrock	3.10^8	15 min
Rosenbrock	3.10^9	148 min
Step	3.10^8	12 min

linearly and hardware and software speed appears as potential constraints. To improve capabilities of modern search methods time consuming events should be identified and optimised. For further investigation on high dimensional problems hardware speed should be improved. Regarding the time required to resolve multidimensional task with acceptable level of precision, presented on Table 2 results suggest that on used hardware configuration selected 300 dimensional tests could be resolved with high probability for the range of 0.5 to 3.5 hours. For more general conclusion additional experiments with 300 dimensional tests should be done.

Comparison on 100 [7], 200 [8] and 300 dimensional tests performance indicates that:

(1) Complexity of task specific landscapes varies among the tests and for same dimensionality different number of functions evaluations could guarantee successful results. This is illustrated with Tables 1, 2, and 3 with Rosenbrock test function.
(2) Test complexity increases nonlinearly to test dimensionality and higher number of functions evaluations are required to reach the same level of precision and standard deviation.

According to results published earlier on Michalewicz test for 100 dimensions for 10^8 objective function evaluations Free Search reaches 0.00048003 standard deviation [7], for 200 dimensions for 2.10^8 objective function evaluations Free Search reaches 0.001784807 standard deviation [8]. In this investigation for 300 dimensions 3.10^8 objective function evaluations Free Search reaches 0.00325299 standard deviation (Table 4). The results suggests that although the number of objective function evaluations is proportional to the number of dimensions, achieved standard deviation tends to decrease. For higher precision additional objective function evaluations are required.

In summary presented results suggest that search process orientation based on heuristic trial and error could cope with multidimensional space. For more general conclusion additional investigation should be done.

Table 4. Performance on Michalewicz test for 100, 200 and 300 dimensions

Michalewicz test	Function evaluations	Maximal	Mean	Deviation
100 dimensions	100 000 000	99.6191	99.618175	0.000480030
200 dimensions	200 000 000	199.612	199.608409	0.001784807
300 dimensions	300 000 000	299.603	299.595365	0.003252990

7 Conclusion

This article presents experimental evaluation of Free Search on 300 dimensional tests. Identified are specific issues related with multidimensional optimisation. Experimental results are also summarized and analysed. Further investigation could focus on evaluation and measure of the time and computational resources sufficient for completion of other multidimensional tasks using parallel processing systems or parallel implementation of the method, which uses several processor cores in parallel or apply accelerated processing based on Graphics Processing Unit (GPU). Algorithms analysis and improvement could be also subject of future research.

Acknowledgements. I would like to thank to my students Adel Al Hamadan, Asim Al Nashwan, Dimitrios Kalfas, Georgius Haritonidis, and Michael Borg for the design, implementation and overclocking of desktop PC used for completion of the experiments presented in this article.

References

1. Ackley, D.H.: A Connectionist Machine for Genetic Hillclimbing. Kluwer, Boston (1987)
2. Censor, Y.: Optimisation Methods, Encyclopedia of Computer Science. Nature Publishing Group, London (2000). pp. 1339–1341
3. De Jung, K.A.: An analysis of the behaviour of a class of genetic adaptive systems. Ph.D thesis, University of Michigan, USA (1975)
4. Griewank, A.O.: Generalized decent for global optimization. J. Optim. Theor. Appl. **34**, 11–31 (1981)
5. Michalewicz, Z.: Genetic Algorithms + Data Structures = Evolution Programs. Springer, Heidelberg (1992)
6. Penev, K.: Free search of real value or how to make computers think. St. Qu, UK (2008). ISBN 978-0-9558948-0-0
7. Penev, K.: Free search – comparative analysis 100. Int. J. Metaheuristics **3**(2), 118–132 (2014)
8. Penev, K.: Free search in multidimensional space II. In: Dimov, I., Fidanova, S., Lirkov, I. (eds.) NMA 2014. LNCS, vol. 8962, pp. 103–111. Springer, Heidelberg (2015)
9. Rosenbrock, H.H.: An automate method for finding the greatest or least value of a function. Comput. J **3**, 175–184 (1960)

kalin.penev@solent.ac.uk