

# Free Search in Multidimensional Space

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# Aims

- Evaluation on multidimensional tests of
  - Free Search
  - Differential Evolution
  - Particle Swarm Optimisation
- Study abilities to avoid stagnation and trapping in local suboptimal solution
- Identify minimal number of iterations and time required to resolve multidimensional tasks with acceptable precision

# Test Problems

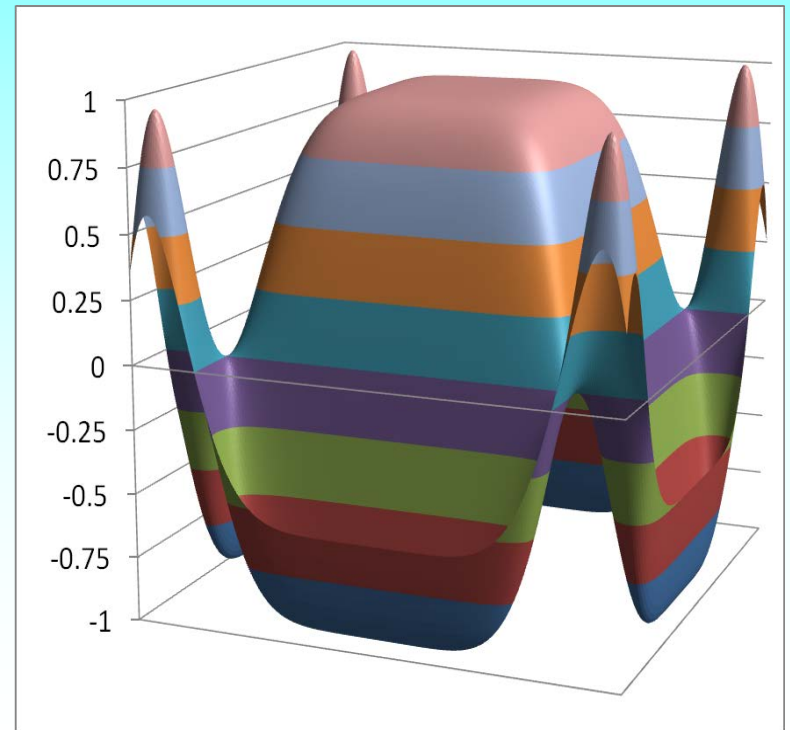
- Criteria for tests selection:
- must be global optimisation test with many local suboptimal solutions;
- must not provide initial knowledge for optimal solution value and location;
- optimal solution must be dependent on dimensions number.

# Test problems

Norwegian test

$$\prod_{i=1}^n \left( \cos(\pi x_i^3) \left( \frac{99 + x_i}{100} \right) \right)$$

where  $x_i \in [-1.1, 1.1]$  for  $i=1, \dots, n$

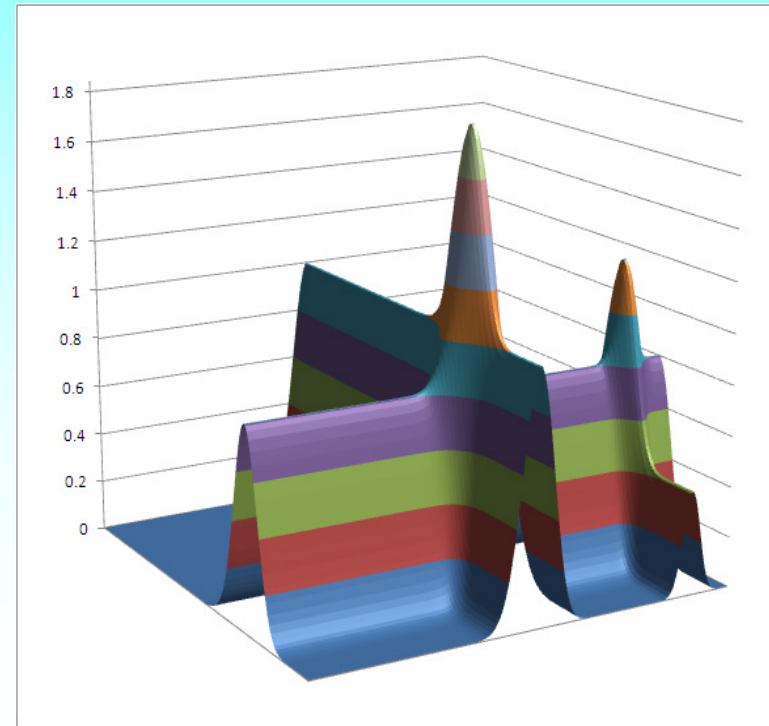


# Test problems

Mochalevicz test

$$f(x_1, x_2) = \sum_{i=1}^2 \sin(x_i) (\sin(ix_i^2 / \pi))^{2m}$$

where  $x_i \in [0.0, 3.0]$  for  $i=1, \dots, n$



# Test problems

## Bump test

$$f(x_i) = \left| \sum_{i=1}^n \cos^4(x_i) - 2 \prod_{i=1}^n \cos^2(x_i) \right| / \sqrt{\sum_{i=1}^n i x_i^2}$$

for:  $0 < x_i < 10$  and  $i=1, \dots, n$

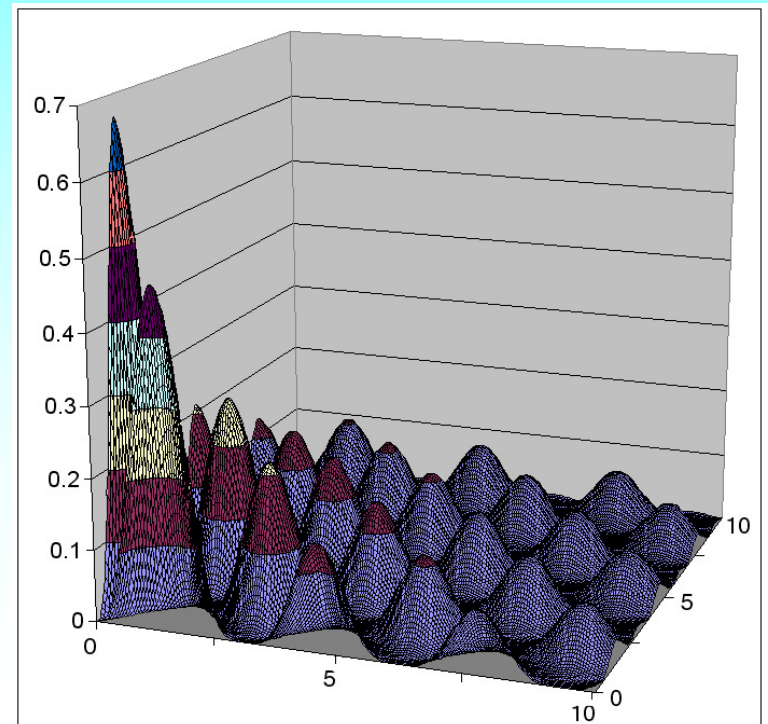
subject to:

$$\prod_{i=1}^n x_i > 0.75,$$

$$\sum_{i=1}^n x_i < 15 \cdot n / 2, \quad i=1, \dots, n$$

starting from one location

$$x_i = 5, \quad i=1, \dots, n$$



# Particle Swarm Optimization

## ❖ Modification strategy

- $$V_{id} = w * V_{id} + n_1 * random(0,1) * (P_{id} - X_{id}) + n_2 * random(0,1) * (g_d - X_{id})$$

- $$X_{id} = X_{id} + V_{id}$$

$w$  is inertia weight

$v$  - velocity vector

$n_1$  is individual learning factor

$n_2$  is social learning factor

$g$  - best achievement for all population

$X_i$  - particle current position

$P_i$  - the best particles' achievement



# Differential Evolution

## ❖ Modification strategy

- (1)  $X'_k = X_k + F(X_i - X_j),$
- (2)  $X'_k = X^* + F(X_i - X_j),$
- (3)  $X'_k = X_k + F(X^* - X_k) + F(X_i - X_j),$
- (4)  $X'_k = X^* + F(X_i - X_j + X_n - X_m),$
- (5)  $X'_k = X_k + F(X^* - X_k + X_n - X_m),$

$X_k$  is a donor vector

$X'_k$  is mutated donor

$X^*$  is the best vector for current population

$X_i, X_j, X_n$  and  $X_m$  are differential vectors

$F$  is differential factor

# Free Search

## ❖ Modification strategy

$$X' = X + R * \text{Rand} * (X_{\max} - X_{\min})$$

X is an initial location

X' is a new location

R - neighbour space

Rand - random value between 0 and 1

Xmax & Xmin - search space limits

# Experimental methodology

- 320 experiments - start from different initial locations
- 3 series for each Norwegian and Michalewicz test limited to:
  - 100000 iterations
  - 1000000 iterations
  - 10000000 iterations
- for Differential Evolution, Particle Swarm Optimisation and Free Search.
- Free Search exploration is 5 steps  
10 iteration = 2 explorations \* 5 steps
- Population size 10 for PSO, DE and FS
- Number of dimensions -100

# Experimental results

- Maximal results on Norwegian and Michalewicz tests

	Iterations	FS	DE	PSO
F1	100 000	0.750627	0.448729	0.220553
	1 000 000	0.967082	0.448729	0.224411
	10 000 000	1.00004	0.490885	0.225525
F2	100 000	99.5808	82.1164	79.2948
	1 000 000	99.6157	87.4321	79.2948
	10 000 000	99.6191	88.2164	79.2948

# Experimental results

- Mean results on Norwegian and Michalewicz tests

	Iterations	FS	DE	PSO
F1	100 000	0.69120580	0.12018652	0.00747217
	1 000 000	0.92401155	0.20175355	0.00798572
	10 000 000	0.98937421	0.2493765	0.00836006
F2	100 000	99.5021065	52.1924515	31.9071906
	1 000 000	99.6109537	57.4708753	33.0173021
	10 000 000	99.618175	59.863695	34.2029145

# Experimental results

- Standard deviation on Norwegian and Michalewicz tests

	Iterations	FS	DE	PSO
F1	100 000	0.02712148	0.18597598	0.03091758
	1 000 000	0.01853559	0.20359603	0.03013784
	10 000 000	0.00843936	0.21503870	0.03148975
F2	100 000	0.11389434	19.591272	20.5197324
	1 000 000	0.00266902	19.2625445	20.9821918
	10 000 000	0.00048003	15.9760239	22.3544008

# Experimental results

- Time for one experiment limited to 10 000 000 iterations (2 000 000 explorations \* 5 steps for FS) for all methods this corresponds to 100 000 000 objective function evaluations (OFEs)
- F1 - Norwegian test
- F2 - Michalewicz test

	OFEs	FS	DE	PSO
F1	100 000 000	4 min	15 min	37 min
F2	100 000 000	19 min	31 min	55 min

# Experimental results

- Results on Bump test (by 2013)

Dimensions	50	100	200	500	1000
Free Search	0.83526234835811175	0.8456854	0.8506636	0.8512628	0.8514553



# Computer system

- CPU - Intel Core i7-3960X 4.895 GHz
- RAM - G.Skill TridentX 16GB (4x4GB) DDR3 1866MHz
- Motherboard - ASUS Rampage IV Extreme 2011
- SSD - SanDisk Extreme SSD SATA III

# Further directions

- Higher dimensions tests evaluation
- Exploration of other tests
- Search methods improvement
- Application to real world problems

Thank you